Graphing Calculators and Their Effect on Students' Understanding of Functions

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Abstract: Scientific graphing technology offers teaching and learning aids currently utilized mainly by teachers and students in upper-level mathematics classes. In this study, a class of ninth-grade Algebra I students possessing only basic scientific calculator experience used scientific graphing calculators to aid in graphing linear equations and inequalities. Results from the study confirmed that the students' use of graphing calculators positively influenced their ability to graph and identify graphs of linear equations. Students who scored higher on the pretest, indicating stronger initial graphing skills, tended to use the calculator to check their work. Students who did not score well on the pretest tended to use the calculator to graph the equation and then copy the results onto their graph paper. Overall, students confirmed that they were more comfortable with using the graphing calculator after the intervention. Previous studies had shown that graphing calculators were viable tools for higher level mathematics classes in high school. This study presented evidence that this tool was just as beneficial for ninth-graders taking Algebra I.

Introduction

One of the main expectations of students in grades 9-12, according the National Council of Teachers of Mathematics (NCTM) as listed in "Principles and Standards for School Mathematics", is that "students' algebra experience should enable them to create and use tabular, symbolic, graphical, and verbal representations to analyze and understand patterns, relations, and functions with more sophistication than in the middle grades" (NCTM, 2000, p. 297). The NCTM (2000) also points out that the use of representations has become more important than in the past in both mathematics and science education. Research, personal experiences, and reports from other high school mathematics teachers suggest to me that many students experience difficulty in understanding functions and graphical representation. The difficulty hinders the students' ability to analyze and understand the patterns represented by the graphs of functions.

The study of functions and their corresponding graphs is an integral part of the concepts Algebra I teachers strive to teach every student. Personal experience and a review of the literature reveals that in this new technology-rich millennium, algebraic concepts, including functions and their graphs, are commonly taught using the basic paper and pencil method. There has been little change in the teaching methods from thirty years ago when the first electronic calculators were made available to the public. While over one-third of all schools surveyed by Dion, et al. (2001) reported that usage of non-graphing calculators including four-function, fraction, and scientific models was encouraged beginning in ninth grade, full integration of these calculators into the curriculum was not readily apparent. Few mathematics unit tests contained questions for which a calculator was required. The use of the more sophisticated scientific graphing calculators, commonly called graphing calculators, which are in essence miniature computers, is even less apparent in all but the higher level mathematics classes in most high schools (Beckmann, Senk, & Thompson, 1999). And yet, the research asserts that where graphing technology was used, students were better able to read and interpret graphs and relate graphs to their equations in addition to being better problem solvers (Dunham, 1998). The NCTM (2000) strongly encourages the use of technology to aid students in learning all aspects of mathematics and specifically advocates the use of computer programs and graphing calculators within the classroom. In spite of the research, many high school mathematics teachers continue to debate whether this technology is appropriate in their classrooms (Dunham).

The use of those early calculators ignited the debate within the educational community and throughout the world, which rages on today (Ellington, 2003). Early debates centered on the use of calculators as an educational tool. As calculator technology became more sophisticated and easier to obtain, the scientific non-graphing calculator became an accepted and often required tool in most American high school mathematics classrooms, causing the debate to change (Milou, 1999). The question now debated is how this tool can be used most effectively to benefit students (Jeffcoat, 2005). While any type of calculator use in the classroom is still debated, the debate is centered on the use of scientific graphing calculators, particularly in Algebra I classes (Milou).

A majority of experimental studies on graphing calculator usage in upper level mathematics class, typically Algebra II, Algebra III, and Pre-calculus classes, have found that graphing calculators have had a positive impact on students' achievement (Dunham, 1998; Ellington, 2003; Milou, 1999). Researchers believe that in order to encourage more Algebra I teachers to take advantage of the potentially powerful teaching aid, more research is needed which demonstrates the usefulness and the viability of integrating scientific graphing calculators into the curriculum (McCauliff, 2004; Milou). This study was conducted to determine if the use of scientific graphing calculators in teaching a unit on graphing linear equations and inequalities would have an effect on ninth-grade Algebra I students' opinions and knowledge of the graphing of algebraic linear functions.

In addition, I was interested in exploring whether students would be able to explain how changes in the coefficient of a variable would affect the graph of the function. The coefficient of a variable is defined as the number in front of the variable. I wanted to determine if the students gained confidence in their ability to describe graphs of certain functions. I also wanted to answer these questions: How do students interact with the graphing calculator as they learn about the graphs of functions? What are the teacher's perceptions towards the use of this technology?

Method

School Setting and Participants

The study took place in a high school located approximately 40 miles from a major metropolitan area. This school, originally built in the mid-1970s for 1750 students, currently contains an excess of 3000 students in grades nine through twelve. The school, like its surroundings is rapidly evolving from a rural setting into a suburban one. The county in which the school is located is categorized as one of the fastest growing counties in the United States. Currently the county's school system is comprised of twelve elementary schools, four middle schools, two high schools and an alternative school. A third high school is scheduled to open in July 2006.

The study was conducted in an Algebra class I was assigned to teach this semester. The intervention class consisted of twenty-four first-time ninth graders. The ethnic breakdown included 9 African-American females, 5 Caucasian females, 5 African-American males, 3 Caucasian males, 1 Asian female, and 1 Inter-racial male. The students were given passive parental permission forms as well as individual permission forms. Two of the students, 1 African-American male and 1 African-American female did not participate in the study. The male did not participate at his parents' request and the female did not participate because of an illness that precipitated her absence during the study. Another Algebra I class I taught was used for the baseline data for test score comparison only. This class of 17 ninth-grade students had an ethic breakdown of 3 African-American females, 2 Caucasian females, 6 African-American males, 4 Caucasian males, 1 Hispanic female, and 1 Inter-racial male.

Intervention

Linear equations and the graphing of linear equations are traditionally taught using paper and pencil. This semester, I obtained 12 TI-81, graphing calculators, and 1 overhead TI-81 for use in one of my Algebra I classes. The graphing calculators were used in place of the paper and pencil method I used in my other classes. All of the students, in both the intervention and traditional classes, worked in cooperative pairs to facilitate their exploration and understanding of the unit. The intervention class shared calculators as sufficient calculators were not available to provide each student with a calculator. The one student who did not participate worked on the assignment without the benefit of using a graphing calculator as did the students in my other three classes.

Measures

Pretests and posttests were used to measure gains in knowledge. Student interaction with the graphing calculator was observed using a checklist. Notations of students' comments both positive and negative were recorded on this checklist form. Pre-intervention and post-intervention surveys were used to measure changes in students' opinions. Since I was also interested in learning whether I had changes in my personal opinions concerning the use of graphing calculators, I kept a journal prior to and during the intervention in hopes of documenting my ideas and opinions.

Procedures

The participants had completed part one of the two-semester Algebra I course required as part of their program of study in our school system. This meant that each of the students had received instruction on graphing linear equations prior to the beginning of the intervention. The second semester in Algebra I, our curriculum requires that we take students from solving linear equations using graphing to solving systems of equations using graphing as one of three methods we teach. I typically begin this unit by reviewing the basics of graphing a linear equation with my students, then progress from that point into the graphing of systems of equations.

Students in the intervention class and the traditional classes took a pretest on the first day to test their knowledge on graphing linear equations. The test consisted of 12 problems. Four problems of the 12 tested their ability to graph a linear equation written in slope-intercept form, two tested their ability to define the direction of the graph of an equation, and six tested their ability to match graphs to their linear equation. After completing the test, the intervention class answered the pre-intervention survey questions.

The intervention class received instructions on how to use the graphing calculator on the second day. Students were allotted time to explore the graphing of functions. Each group of students received a list of activities to perform using the graphing calculator. An observation of student interaction with the graphing calculators was made as the students completed the activities. The other classes received standard review information.

On day three of the study, all of the classes received graph paper and a list of equations to graph. The intervention class used the graphing calculators in addition to the graph paper. During this activity, I observed the students as they interacted with the graphing calculator. Students in all classes worked with a partner.

On day four, I provided the students with a list of linear inequalities to graph. After a brief discussion on the similarities and differences in graphing these inequalities, they worked with their partner to graph the inequalities. The students in the intervention class used a graphing calculator in addition to graphing the inequalities onto graph paper. On day five, the students were given a variety of equations and inequalities to graph as a review. I completed an observation of the students in the intervention class on days three through five. All equalities and inequalities were presented in slope-intercept form to insure that an inability to rearrange an equation or inequality was not a factor in their graphing activity.

On day six, all students took the posttest. The posttest was identical to the pretest. Graphing calculators were not used for the posttest. After the test, the students in the intervention class completed the post-intervention survey.

Results

In the pre-intervention survey, the first nine questions were asked to determine what access students had to calculators and their perceived individual comfort level in using these calculators. Seventy-three percent of the students reported that they had access to a scientific calculator at home. Fifty percent reported that they had used a scientific calculator for less than a year, while twenty-seven percent had used a scientific calculator for one to three years and twenty-three percent had used a scientific calculator for more than four years. Only one of the students reported having access to a scientific graphing calculator at home, and only three of the students had ever used a scientific graphing calculator before.

Sixty-eight percent of the students agreed and thirty-one percent strongly agreed that they were comfortable with using a scientific calculator to perform standard calculating functions. Ninety-one percent of the students agreed or strongly agreed that they scored better on math tests when they were allowed to use a calculator. Fifty-nine percent agreed or strongly agreed that they used a calculator to calculate all mathematical computations while forty-one percent disagreed that they used calculators to perform all computations. Seventy-seven percent of the students responded that they agreed or strongly agreed that they regularly use non-graphing calculators in their Algebra I class.

The last ten items, created to measure changes in opinions, were similar to the ten questions on the post-intervention survey. A comparison of the answers is given in Table 1 below. Student responses were measured using a Likert scale. Students marked 1 if they strongly disagreed, 2 if they disagreed, 3 if they agreed, and 4 if they strongly agreed. Overall student responses showed some increased confidence in their ability to use the graphing calculator to calculate numbers and graph linear equations. Student preference to using graph paper increased slightly from a mean of 1.77 to 2.09, indicating less aversion to the traditional method of graphing functions after the intervention. The students originally indicated that they believed they would enjoy this unit because they would use a graphing calculator (M = 3.27); but after the intervention that mean dropped to a 2.91, indicating less enthusiasm.

Comparison of the pre-intervention survey and the post-intervention survey indicated increased student enjoyment in using the graphing calculators to explore graphs. Students also exhibited an increased comfort level in using the graphing calculators to perform numerical calculations. Before the intervention, students indicated that they agreed that they would score higher on this unit test; after the intervention students disagreed with this statement.

Questions 1 = strongly disagree, 2 = disagree, 3 = agree, and 4 = strongly agree	Pre		Post	
	M	SD	M	SD
I am comfortable using a graphing calculator to calculate numbers.	3.18	.50	3.23	.69
I am comfortable using a graphing calculator to graph linear equations.	2.91	.75	3.18	.91
I prefer using graph paper to draw my own graphs instead of using the graphing calculator to graph linear equations.	1.77	.75	2.09	.92
I enjoy(ed) using a graphing calculator to explore graphs of equations	2.76	.70	3.05	.84
I believe that I learn more when I am taught in a traditional method (teacher talking, worksheets, and book) without the use of calculators.	2.18	.91	2.32	.84
I prefer to work on my own rather than in a group to complete projects involving calculators.	2.23	.92	2.05	1
I believe that I benefit(ed) from working in groups or with a partner on this unit.	2.91	.75	2.73	.94
I (will score) scored a higher grade on this Algebra I test than I scored on other tests because I used a graphing calculator.	2.95	.84	2.36	.90
This unit will be (was) more difficult than most Algebra I units because I was required to use a graphing calculator.	2.18	.66	2.14	.94
Overall, I will enjoy (enjoyed) this unit on linear equations because I am (was) able to use a graphing calculator as part of the learning process.	3.27	.63	2.91	.97

Table 1: Results of Student Survey, Pre- and Post-intervention (n =22)

The pre- and post-intervention surveys included open-ended questions. In the pre-intervention surveys, I asked the students if they believed that increasing graphing calculator usage in class would help them learn the graphing concepts better. The students overwhelmingly responded positively to this question. One student wrote, “We ought to be able to use computers not just calculators to do some of this stuff for us.” Most students responded that they had only used calculators in the past to calculate numbers. The answer given most often to the question on what students thought teachers could do to make Algebra I classes more interesting was that “math is boring and teachers should make it (Algebra) ‘more fun’.”

The post-intervention open-ended questions asked students if they thought that the graphing calculator helped them learn the concepts. Nine of the 22 students said that the calculator was not helpful to them, while many of the other students said that using the calculator helped them because it was, “more hands on.” Some students commented that learning to operate the calculator was “not hard”, but one student said that, “using the calculator confused him.” The last question asked students what they thought could be done to make the unit better. Some of the students stated that they would have liked to have one calculator per student. Some mentioned wanting more time to learn the calculator and the need for more examples. Three of the students suggested more one-on-one time with the teacher would help them more than using a graphing calculator.

The pretest and posttest scores, as shown in Table 2 below, indicated an average 19.7 point increase in test scores for the intervention class while the traditional class showed only a 7.41 point increase in test scores. However, the students in the intervention class scored much lower on the pretest than the students in the traditional class.

Intervention Class Scores (n = 22)				Traditional Class Score (n = 17)			
Pretest		Posttest		Pretest		Posttest	
M	SD	M	SD	M	SD	M	SD
53.73	33.38	74.64	25.41	73.06	26.70	80.47	32.79

Table 2: Pretest and Posttest Results

The comparisons in Table 3, shown below, include a breakdown in the test scores by gender for each type of question, as well as a breakdown by gender of overall test scores. There were fourteen females in the intervention class compared to six females in the traditional class. The females in the intervention class scored below 55% in each area of the pretest, while the females in the traditional class scored 67% or above in each area of the pretest. Females in both classes showed a gain in their posttest scores, but the females in the intervention class had the highest overall gain.

The eight males in the intervention class averaged scores of 73% on the pretest while the eleven males in the traditional class averaged scores of 72%. While the overall scores were similar, there were differences in results by question type as shown in Table 3. In the intervention class, males graphed 78% of the linear equations correctly in the pretest, and 88% in the posttest. Males in the traditional class correctly graphed only 61% of the linear equations in the pretest, and 84% in the posttest. The intervention class males correctly matched graphs to their equations 65% in the pretest, 94% in the posttest. The traditional class males correctly matched graphs to their equations 73% in the pretest, 82% in the posttest.

Table 3 also shows that while all of the students in the intervention class demonstrated an increased ability to graph and match graphs to their linear equations after the intervention, there was a slight drop in the ability to identify the direction of the slope of the line. The students in the traditional class also demonstrated an increase in their ability to graph a function and match the graph to its corresponding function, as well as an increased ability to identify the slope direction of a linear function.

Intervention class (n = 22)						
Question Type	**Pretest Scores**			**Posttest Scores**		
	Class	Females (n = 14)	Males (n = 8)	Class	Females (n = 14)	Males (n = 8)
Graph the equation: Percent correct	47%	29%	78%	69%	59%	88%
Match the equation to its graph: Percent correct	58%	54%	65%	85%	80%	94%
Is the slope positive or negative? Percent correct	70%	54%	100%	66%	54%	88%
Entire Test	Class	Females	Males	Class	Females	Males

		(n = 14)	(n = 8)		(n = 14)	(n = 8)
Rounded to nearest whole percent	54%	43%	73%	75%	63%	91%
Traditional class (n = 17)						
	Class	Females (n = 6)	Males (n = 11)	Class	Females (n = 6)	Males (n = 11)
Graph the equation: Percent correct	75%	83%	61%	82%	79%	84%
Match the equation to its graph: Percent correct	76%	81%	74%	82%	92%	77%
Is the slope positive or negative? Percent correct	71%	67%	73%	82%	83%	82%
Entire Test	Class	Females (n = 6)	Males (n = 11)	Class	Females (n = 6)	Males (n = 11)
Rounded to nearest whole percent	73%	75%	72%	81%	81%	80%

Table 3: Pretest and Posttest Results by Question Type and Gender

Analysis of my personal log showed that at the beginning of the study I was skeptical but hopeful, that graphing calculators would help my students understand graphs of linear functions. My students had not performed very well on the pretest and their feedback during the first class was not very positive. The students' reactions became more positive as they worked more with the calculators and started being able to predict how the function would look when graphed. At this point, my notes at the end of class became more upbeat. During the unit, I often felt rushed as I worked to incorporate the calculator into my instruction, and at one point, I noted that I wasn't sure if it was worth the effort.

Discussion

I set out to discover if the use of graphing calculators would enable ninth-grade Algebra I students to improve their knowledge of the relationship that exists between functions and their graphs. Positive results reported in previous studies with students in higher level mathematic classes had suggested that this technology if utilized would prove beneficial to Algebra I students. I found that the use of graphing calculators did seem to have a positive impact on my students' knowledge of graphing linear equations and inequalities. Overall, the students in both the intervention and the traditional classes demonstrated increased scores on the posttest. The intervention class scores improved approximately 21% while the traditional class scores improved approximately 7.5%.

The large disparity in the class gains may be partly attributed to differences in the number of males and females in the two classes. The intervention class was comprised of 63.6 % female and 36.4% male, while the traditional class was 35.3% female and 64.7% male. The pretest average for the intervention class was 19% points lower than the pretest average in the traditional class. A comparison of gender scores showed that the males in both classes scored similar averages on the pretest. Males in the intervention class averaged 72.88% on the pretest, while males in the traditional class averaged 71.91% on the pretest. Comparisons of the females' pretest scores were not as similar. Females in the intervention class averaged 42.79% on the pretest, while their counterparts in the traditional class averaged 75.17% on the pretest.

The average scores on the posttest do show that the graphing calculator usage proved advantageous for all students in the intervention class. Females in the intervention class improved their overall test averages by 20.64%. Females in the traditional class improved their averages by 5.5%. Males in the intervention class averaged a 17.63% gain in grades on the posttest, while the males in the traditional class averaged an 8.45% gain on their posttest scores. One interesting point is that although the students in the intervention class increased their ability to match functions with their graphs and their ability to draw graphs, they did not increase in their ability to identify which way the slope was leaning. The lack of increased understanding about the direction in which the slope was leaning may be attributed to lack of a clear definition of the terminology. Students may not have understood exactly what the question was asking.

At the beginning of the study, I found that initial student excitement gave way to frustration as students tried to learn how to use the graphing calculator while having to graph functions. The students followed the guided reflection for the first equation or two and very quickly grew weary of trying to follow the outline. They preferred simply graphing the functions. I noted in my observations that most students did not show interest in exploring the functions using the graphing calculators. When students were questioned about this behavior, they complained about the amount of work that it took to follow the guided reflection. Some students said following the guided reflection, "felt stupid." Although students performed better on the posttest, many claimed that they still did not understand graphs. This may indicate that the guided reflection needs to be evaluated and revised to allow for less repetitive questions. A better approach might be to give the students a list of equations, some of which are identical except for the slope or the y-intercept. The students could then graph each on the calculator and then answer just a question or two about each graph as they view that graph of the function on the calculator. This approach would help guide the exploration in a less structured and demanding manner.

The manner in which students interacted with the calculator was interesting. I observed some students graphing their equations on graph paper and then using the calculator to check their graph, indicating that these students were more confident in their ability. Other students would use the graphing calculator to graph the function and then transfer the results onto graph paper, indicating that they were not very confident in their ability to graph a function without the calculator. The males tended to use the calculator more to check their work. The females, on the other hand, often used the calculator to graph the function and copied the resulting graph onto the graph paper.

On average, the males in the intervention class scored better on the pretest than the females in this class. This might indicate that the males were more comfortable with graphing functions before the study. The females in the intervention class showed the biggest increase in understanding of functions and their graphs after using the graphing calculators. In the intervention class, the largest gain in understanding appeared to be in the students' ability to match graphs to their corresponding functions, indicating that the graphing calculators can help students gain a better understanding of the relationship that exists between the graph and the function.

After conducting all of the research and reading several positive research studies, I was hopeful that the intervention would be a positive experience for the students. The students did indicate that they enjoyed using a graphing calculator more than they originally thought they would. Some students expressed dissatisfaction with the amount of time allotted to work with the calculators. Others remarked that sharing a calculator hindered their ability to learn how to use the calculator. A few students stated that having to learn how to use the calculator created more work. They had hoped that the calculator would make their work easier! The students disliked that they were not allowed to use the calculator on the posttest.

At the beginning of the study, I was hopeful that the students would benefit from the exposure to this technology. I was not prepared for all of the work involved in using these graphing calculators. My lesson plans were very ambitious and complex. Simpler lesson plans would have helped make the lessons more enjoyable for the students and for me. I realized early in the study that I had not allotted enough time for student familiarization with the calculator. Giving the students time to become acclimated to the calculators prior to instruction might have given the students more confidence in using the calculators. My reflective journal chronicled my inexperience in using this tool in my instructional unit. I was much more comfortable teaching my other classes using traditional methods.

I believe that this study demonstrated that the use of graphing calculators as an instructional aid does help students increase their knowledge of graphing concepts. Use of the graphing calculators appeared to strengthen the females' grasp of the concepts, suggesting that this tool would be useful in helping females be more successful in mathematics classes. More time should have been spent to ensure that students understood how to use the calculators prior to actually using them within a lesson. The short duration of the study only whetted my curiosity and the results strengthened my belief that simply allowing the calculators to be used in class is not enough. Future lesson plans must include built-in activities that take advantage of the power and capabilities of the calculator.

There was a lack of equivalence between classes due in part to the difference in the number of females in the two classes. The number of males in both classes was very similar and they scored similar averages on the pretest. The disparity recorded in the average scores of the females on the pretest may have been caused by the higher ratio of females in the intervention class. Future studies should attempt to match up the gender ratios more equivalently.

Overall, the study results in this Algebra I class agree with the result of studies conducted in more advanced mathematics classes. The results justify the need for a longer study that would explore what it is about the graphing calculator that helps improve the understanding of functions and graphs. This short study needs to be followed up by a longer study on how prolonged use of the graphing calculator in Algebra I classes helps student gain a better knowledge of graphing functions. This study showed that using the graphing calculator yields positive short-term results. But does it produce long-term results? This question should be explored.

The interest generated by this action research culminated in a celebration attended by the mathematics teachers at my school. Preliminary results of the study were provided to attendees via a brief oral report accompanied by transparencies containing charts and graphs displaying these results. In the question and answer session following the presentation, many of the attending teachers expressed interest in the results and commented on the positive changes technology generally fosters in students. Several teachers remarked that the lack of sufficient graphing calculators makes it difficult to incorporate the technology into more of their lessons. In addition, teachers suggested that staff development classes on ways to incorporate graphing calculators into lesson plans be offered to mathematics teachers in the future. These suggestions coupled with a copy of the results of the study were presented to administrators who were unable to attend the celebration.

The use of computerized technology such as graphing calculators is used increasingly in our lives outside the classroom. It is up to the educational community to insure that this technology is also available and fully utilized within the classroom.

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