

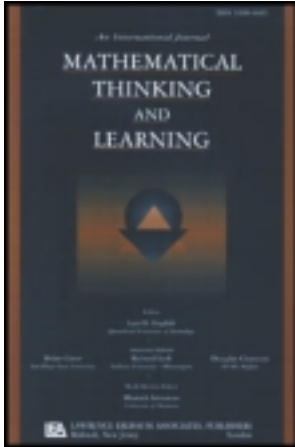
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# A Situated and Sociocultural Perspective on Bilingual Mathematics Learners

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My aim in this article is to explore 3 perspectives on bilingual mathematics learners and to consider how a situated and sociocultural perspective can inform work in this area. The 1st perspective focuses on acquisition of vocabulary, the 2nd focuses on the construction of multiple meanings across registers, and the 3rd focuses on participation in mathematical practices. The 3rd perspective is based on sociocultural and situated views of both language and mathematics learning. In 2 mathematical discussions, I illustrate how a situated and sociocultural perspective can complicate our understanding of bilingual mathematics learners and expand our view of what counts as competence in mathematical communication.

Language-minority students remain severely underrepresented in technical and scientific fields (Secada, 1992), there are wide gaps between the performances of White and Latino students (Educational Testing Service, 1991), and mathematics courses in middle schools and high schools continue to function as a “critical filter” for some Latino students (Oakes, 1990). Although the educational reform movement has attempted to address the needs of language-minority students, it may be leaving a substantial number of these students unaffected (Gándara, 1994; Valadez, 1989). If mathematics reforms are to include language-minority students, research needs to address the relation between language and mathematics learning from a perspective that combines current perspectives of mathematics learning with current perspectives of language, bilingualism, and classroom discourse. This research can then facilitate the design of curriculum and instruction that will address the needs of language-minority students and ultimately support the success of these students in mathematics.

Despite the steadily increasing population of U.S. students who are classified as limited in English proficiency (estimated to be 5 million with 1 million of these in California, mostly Latinos), there has been little research addressing these students' needs in mathematics classrooms. Early studies of bilingual students learning mathematics framed the challenges that bilingual Latino students faced in terms of solving word problems, understanding vocabulary, or translating from English to mathematical symbols (Cocking & Mestre, 1988; Cuevas, 1983; Cuevas, Mann, & McClung, 1986; Mestre, 1981, 1988; Spanos & Crandall, 1990; Spanos, Rhodes, Dale, & Crandall, 1988). This focus was reflected in recommendations for mathematics instruction for English-language learners that emphasized vocabulary and comprehension skills (Dale & Cuevas, 1987; MacGregor & Moore, 1992; Rubenstein, 1996).

In contrast, more recent research on mathematics learning has focused on how students construct knowledge, negotiate meanings, and participate in mathematical communication. Although several studies have focused on these issues in monolingual mathematics and science classrooms (Cobb, Wood, & Yackel, 1993; Forman, 1996; Lemke, 1990; Pimm, 1987; Pirie, 1991; Richards, 1991), researchers have only recently begun to consider mathematical communication in language-minority classrooms (Adler, 1998; Brenner, 1994; Khisty, 1995; Khisty, McLeod, & Bertilson, 1990; Moschkovich, 1999; Rosebery, Warren, & Conant, 1992; Thornburg & Karp, 1992). Although these studies are a beginning step, there is a need for more research on how language-minority students learn mathematics from a perspective that includes mathematical communication as an integral aspect of learning mathematics.

Mathematics curriculum and teaching standards have come to reflect this current model of mathematics learning that emphasizes communication (National Council of Teachers of Mathematics, 1989). As mathematics classrooms shift from a focus on primarily silent and individual activities (Cazden, 1986; Stodolsky, 1988) to more verbal and social ones (Cobb et al., 1993; Flores, Sowder, Philipp, & Schapelle, 1996; Forman, 1996), bilingual Latino students face new challenges and opportunities in learning mathematics. In reform-oriented mathematics classrooms, students are no longer grappling primarily with acquiring technical vocabulary, developing comprehension skills to read and understand mathematics textbooks, or solving traditional word problems. Students are now expected to communicate mathematically, both orally and in writing, and participate in mathematical practices, such as explaining solution processes, describing conjectures, proving conclusions, and presenting arguments.

Some concerns that stem from these reforms are how bilingual Latino students will be affected by this emphasis on mathematical communication and how classroom instruction can support these students in learning to communicate mathematically. In this article, I address these concerns by proposing and exploring three perspectives for describing mathematics learning and its relation to language. The

first perspective emphasizes vocabulary and describes learning mathematics as the acquisition of vocabulary. The second perspective emphasizes word meanings, uses the concept of registers (Halliday, 1978), and describes learning mathematics as the construction of multiple meanings across the everyday and mathematical registers. The third perspective emphasizes the situated<sup>1</sup> and sociocultural nature of language and mathematics learning, uses the concept of Discourses as defined by Gee (1996, 1999), and describes learning mathematics as participation in mathematical Discourse practices. An important practical implication of this situated–sociocultural perspective is that it allows us to more fully describe the variety of resources that students use to communicate mathematically.

In this article, I first summarize the main features of these three perspectives, detailing how a situated–sociocultural perspective can inform the study of mathematical communication in classrooms with bilingual students. Because of their critical importance to the distinctions I make between these three perspectives, I also briefly describe how concepts such as register, discourse, bilingualism, and code switching can be understood from a situated–sociocultural perspective. Next, I examine excerpts from mathematical discussions in classrooms with bilingual students using these three perspectives. Finally, I argue that a situated–sociocultural perspective complicates our view of how bilingual students learn mathematics and expands what counts as competence in communicating mathematically. A situated–sociocultural perspective is useful for avoiding deficiency models of bilingual learners, developing detailed descriptions of the resources that students use to communicate mathematically, and helping teachers build on these resources during instruction.

### THREE PERSPECTIVES OF BILINGUAL MATHEMATICS LEARNERS

Next, I propose three perspectives for thinking about how bilingual students learn mathematics: acquiring vocabulary, constructing meanings, and participating in discourses. I find these three categories useful for organizing and understanding work on the relation between learning mathematics and language. These perspectives are not meant to represent any one researcher, theorist, or school but are instead offered as composite (and perhaps caricatured) summaries of three theoretical stances I believe are reflected in work in this area. I critique the acquiring vocabulary and constructing multiple-meanings perspectives. My purpose, however, is not to point out how previous work was right or wrong but to examine the

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<sup>1</sup>I use the term *situated* to mean “local, grounded in actual practices and experiences” (Gee, 1999, p. 40). Although *situated* can be understood to mean sociocultural, for the sake of clarity, I use the term *situated–sociocultural*.

limitations we face when using these two perspectives and to describe a progression in terms of the complexity of the phenomena each one includes. I offer the third view, a situated–sociocultural perspective, as an option for complicating how we think about language in learning mathematics and expanding what counts as competence in mathematical communication.

### Acquiring Vocabulary

One way to describe bilingual mathematics learners is that students are acquiring vocabulary. This first perspective defines learning mathematics as learning to carry out computations or solve traditional word problems, and it emphasizes acquiring vocabulary as the central issue that second-language learners are grappling with when learning mathematics. This view of learning mathematics is reflected in the early research on bilingual mathematics learners, which focused primarily on how students understood individual vocabulary terms or translated traditional word problems from English to mathematical symbols (e.g., Mestre, 1981; Spanos et al., 1988). Recommendations for mathematics instruction for English-language learners have also tended to emphasize vocabulary and comprehension skills (Dale & Cuevas, 1987; MacGregor & Moore, 1992; Olivares, 1996; Rubenstein, 1996).

Although an emphasis on vocabulary may have been sufficient in the past, this perspective does not include current views of what it means to learn mathematics. Mathematics learning is now seen not only as developing competence in completing procedures, solving word problems, and using mathematical reasoning but also as developing sociomathematical norms (Cobb et al., 1993), presenting mathematical arguments (Forman, 1996), and participating in mathematical discussions (Lampert, 1990). In general, learning to communicate mathematically is now seen as a central aspect of what it means to learn mathematics.

Research from this perspective has typically been concerned with individual students solving word problems as a paradigmatic case of what it meant to learn mathematics (another paradigmatic case was computation). This perspective may have been useful for describing traditional classroom instruction that focused on solving word problems and individual computation, and it may thus be limited to such cases. Solving word problems was also the prototypical example of how mathematics and language intersect in the classroom. However, in many mathematics classrooms today, students are not grappling primarily with acquiring technical vocabulary, developing comprehension skills to read and understand mathematics textbooks, or solving traditional word problems. (This shift has occurred, in part, because traditional word problems are no longer seen as a paradigmatic case of mathematics learning.) Students are expected to participate in classroom mathematical practices that go beyond solving computation or word problems on a worksheet (Ball, 1991; Forman, 1996; Silver & Smith, 1996). In many classrooms,

teachers are incorporating many forms of mathematical communication, and students are expected to participate in a variety of oral and written practices, such as explaining solution processes, describing conjectures, proving conclusions, and presenting arguments. Reading textbooks and solving traditional word problems are thus no longer the best examples of how language and learning mathematics intersect.

The vocabulary perspective also presents a simplified view of language that uses, perhaps implicitly, the notion of a *lexicon*. A lexicon is “the list of all words and morphemes of a language that is stored in a native speaker’s memory; the internalized dictionary” (Finegan & Besnier, 1989, p. 528). In particular, this perspective does not address the multiple meanings of words. Ultimately, knowing vocabulary is only one part of the story of learning mathematics (or in the case of the following quote, science):

It is not the knowing of a term in and of itself that matters one way or another; rather we found ourselves wondering how we use a particular term, with what intentions, what we are assuming about what others in the conversation might or might not know, and what that term made clear for us or what in fact we understood about the phenomena we had named. (Rosebery & Warren, 1998, p. 7)

An emphasis on vocabulary has crucial implications for instruction. In particular, this perspective can affect how teachers assess a student’s competence in communicating mathematically. For example, if we focus on a student’s failure to use a technical term, we might miss how a student constructs meaning for mathematical terms or uses multiple resources, such as gestures, objects, or everyday experiences. We might also miss how the student uses important aspects of competent mathematical communication that are beyond a vocabulary list.

### Constructing Multiple Meanings

A second perspective for thinking about bilingual mathematics learners describes learning mathematics as constructing multiple meanings for words rather than acquiring a list of words. This perspective uses the notion of the mathematics register. Halliday (1978) defined *register* in the following way:

A register is a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings. We can refer to the “mathematics register,” in the sense of the meanings that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself), and that a language must express if it is being used for mathematical purposes. (p. 195)

A register is a language variety associated with a particular situation of use. Some examples of registers are legal talk and baby talk. An important and subtle distinction is that between lexicon and register. Unlike the notion of lexicon, the notion of register depends on the situational use of much more than lexical items and includes phonology, morphology, syntax, and semantics as well as non-linguistic behavior. The notion of register thus involves some aspects of the situation, whereas that of lexicon does not.

Because there are multiple meanings for the same term, students who are learning mathematics can be described as learning to use these multiple meanings appropriately. Several examples of such multiple meanings have been described: the phrase “any number” means “all numbers” in a math context (Pimm, 1987); “a quarter” can refer to a coin or to a fourth of a whole (Khisty, 1995); and in Spanish, *un cuarto* can mean “a room” or “a fourth” (Khisty, 1995).

These multiple meanings can create obstacles in mathematical conversations because students often use the colloquial meanings of terms, whereas teachers (or other students) may use the mathematical meaning of terms. One example is the word *prime*, which can have different meanings, depending on whether it is used in “prime number,” “prime time,” or “prime rib.” In Spanish, *primo* also has multiple meanings; it can mean “cousin” or “prime number,” as in the phrase “*número primo*.” Another example of multiple meanings is Walkerdine’s (1998) description of the differences between the meanings of *more* in the mathematics classroom and at home. Although in a classroom situation, *more* is usually understood to be the opposite of *less*, at home the opposite of *more* is usually associated with *no more*, as in “I want more paper” and “There is no more paper.”

The multiple-meanings perspective considers differences between the everyday and mathematical registers and describes how students’ language use can move closer to the mathematics register by becoming more precise and reflecting more conceptual knowledge (Forman, 1996; Moschkovich, 1996, 1998; O’Connor, 1992). Learning mathematics involves, in part, a shift from everyday to more mathematical and precise meanings. For example, one important difference between the everyday and the school mathematics registers may be the meaning of relational terms such as *steeper* and *less steep* and phrases such as “moves up the *y* axis” and “moves down the *y* axis.” Meanings for these terms and phrases that may be sufficiently precise for everyday purposes may prove to be ambiguous for describing lines in the context of a mathematical discussion (Moschkovich, 1996).

A refinement of students’ descriptions of mathematical situations can be understood as a movement toward the mathematics register, where descriptions are more precise and reflect more conceptual knowledge. However, the mathematics register does not consist only of technical terms such as *slope* and *intercept*. Students refine their descriptions by connecting even nontechnical phrases such as “the line will be steeper” or “the line will move up on the *y* axis” to conceptual knowledge about lines and equations (Moschkovich, 1998).

Moving across two national languages, for example, English and Spanish, may complicate moving across two registers. For example, the distinction between the following two uses of *más* (“more”) is crucial in a mathematics context:

Hay cuatro más \_\_\_\_\_ que \_\_\_\_\_. [There are four more \_\_\_\_\_ than \_\_\_\_\_.]  
 Hay cuatro veces más \_\_\_\_\_ que \_\_\_\_\_. [There are four times as many  
 \_\_\_\_\_ as \_\_\_\_\_.]

These two sentences refer to two different mathematical situations, and yet the word *más* is used in both cases.

Emphasizing multiple meanings can shift the focus from examining how students acquire vocabulary to asking how students negotiate the meaning of mathematical terms. This second perspective shifts our view of learning mathematics from acquiring words to developing meanings for those words, from learning words with single meanings to understanding multiple meanings, and from learning vocabulary to using language in situations. The multiple-meanings perspective certainly adds complexity to our view of how language and learning mathematics intersect. Although a focus on the mathematics register has served to point out possible sources of misunderstanding in classroom conversations, using this perspective also has certain pitfalls that are important to avoid.

First, when using the multiple-meanings perspective, we need to be careful not to interpret the notion of register as a list of technical words and phrases. This interpretation reduces the concept of mathematical register to vocabulary and disregards the role of meaning in learning to communicate mathematically. Second, when using this perspective, we also need to be careful to include the situational context of utterances. Although words and phrases do have multiple meanings, these words and phrases appear in talk as utterances that occur within social contexts, and much of the meaning of an utterance is derived from the situation. For example, the phrase “give me a quarter” uttered at a vending machine clearly has a different meaning than saying “give me a quarter” while looking at a pizza. The utterance “Vuelvo en un cuarto de hora” (“I will return in a quarter of an hour”) said as one leaves a scene has a clearly different meaning than “Limpia tu cuarto” (“Clean your room”), uttered while looking toward a room. When we analyze mathematical conversations, it is important to consider how resources from the situation, such as objects and gestures, point to one or another sense, such as whether *cuarto* means “room” or “quarter.”

A third important limitation of the multiple-meanings perspective is that the differences between the everyday register and the mathematics register are not always a source of difficulty for students. Students use not only mathematical resources but also resources from the everyday register to communicate about a mathematical situation. Forman (1996) offered evidence of this in her description of how students and teachers interweave the everyday and academic registers in classroom



discussions. Similarly, Moschkovich (1996) described how students used the metaphor that a steeper line is harder to climb than a line that is less steep to compare the steepness of lines on a graph and clarify the meaning of their descriptions of lines. One example presented in this article shows how students used another metaphor from everyday experiences, that the ground is the  $x$  axis, to elaborate the meaning of their descriptions of straight lines.

Although differences between the everyday and mathematical registers may sometimes present obstacles for communicating in mathematically precise ways and everyday meanings can sometimes be ambiguous, everyday meanings and metaphors can also be resources for understanding mathematical concepts. Rather than emphasizing the limitations of the everyday register in comparison to the mathematics register, it is important to understand how the two registers serve different purposes and how everyday meanings can provide resources for mathematical communication.

The acquiring vocabulary and constructing multiple-meanings perspectives can have an important impact on instruction. Either of these perspectives can be interpreted as emphasizing the obstacles that bilingual students face as they move from their first language to English or from the everyday register to the mathematics register. Any perspective that focuses on obstacles can easily turn into a deficiency model (Garcia & Gonzalez, 1995; González, 1995) of bilingual students as mathematics learners. The everyday register and students' first language can, in fact, be used as resources for communicating mathematically. Instruction in mathematical communication needs to consider not only the obstacles that bilingual students face but also the resources these students use to communicate mathematically.

Although mathematical communication certainly involves the use of words and constructions and the development of multiple meanings, it is also more than these. Communicating mathematically also includes using multiple resources and participating in mathematical practices, such as abstracting, generalizing, being precise, achieving certainty, explicitly specifying the set of situations for which a claim holds, and tying claims to representations.

### Participating in Mathematical Discourse Practices

The two perspectives summarized previously for describing bilingual students learning mathematics, acquiring vocabulary, and understanding multiple meanings provide some analytical tools for clarifying how bilingual students learn mathematics. In this section, I explore how using the notion of Discourses (Gee, 1996, 1999) and a situated–sociocultural view of mathematics cognition, language, and bilingual learners can provide us with an even more complex and detailed view of bilingual students learning mathematics.

A situated–sociocultural perspective has important implications for instruction. The first two perspectives frame the relation between learning mathematics and

language in terms of the discontinuities between first and second languages or the differences between the everyday and the mathematics register. In contrast, a situated–sociocultural perspective can be used to describe the details and complexities of how students, rather than struggling with the differences between the everyday and the mathematical registers or between two national languages, use resources from both registers and languages to communicate mathematically. A situated–sociocultural perspective thus moves away from the description of obstacles and deficiencies to a description of resources and competencies and widens what counts as competence in mathematical communication.

The situated and sociocultural discourse perspective described here uses a situated perspective of learning mathematics (Greeno, 1994) and the notion of Discourses (Gee, 1996) to build on previous work on classroom mathematical and scientific discourse (Cobb et al., 1993; Lemke, 1990; Rosebery et al., 1992). This perspective implies, first, that learning mathematics (or science) is viewed as a discursive activity (Forman, 1996; Lemke, 1990; Rosebery et al., 1992). From this perspective, learning mathematics is described as participating in a community of practice (Cobb & Hodge, 2002/*this issue*; Forman, 1996; Lave & Wenger, 1991; Nasir, 2002/*this issue*), developing classroom sociomathematical norms (Cobb et al., 1993), and using multiple material, linguistic, and social resources (Greeno, 1994). This perspective assumes that learning is inherently social and cultural, “whether or not it occurs in an overtly social context” (Forman, 1996, p. 117); that participants bring multiple views to a situation; that representations have multiple meanings for participants; and that these multiple meanings for representations and inscriptions are negotiated through conversations.

Situated perspectives of cognition (Brown, Collins, & Duguid, 1989; Greeno, 1994; Lave & Wenger, 1991) present a view of learning mathematics as participation in a community where students learn to mathematize situations, communicate about these situations, and use resources for mathematizing and communicating. From this perspective, learning to communicate mathematically involves more than learning vocabulary or understanding meanings in different registers. Instead, communicating mathematically is seen as using social, linguistic, and material resources to participate in mathematical practices.

To ground the subsequent discussion, I briefly describe how several concepts—*practices*, *bilingualism*, *code switching*, and *Discourses*—are defined from a situated–sociocultural perspective. These notions are not intended to be used as isolated concepts but are meant to be couched within a situated–sociocultural theoretical framework. I use the term *practices* in the sense described by Scribner (1984): “to highlight the culturally organized nature of significant literacy (*or mathematical* [*italics added*]) activities and their conceptual kinship to other culturally organized activities involving different technologies and symbol systems” (p. 13).

Rather than defining a *bilingual learner* as an individual who is proficient in more than one language, I use a situated–sociocultural definition of bilingual

learners as those students who participate in multiple-language communities. As described by Valdes-Fallis (1978), “natural” bilinguals are “the product of a specific linguistic community that uses one of its languages for certain functions and the other for other functions or situations” (p. 4). Work in sociolinguistics has shown that code switching is one of many resources available to bilingual speakers. Code switching is a rule- and constraint-governed process and a dynamic verbal strategy in its own right rather than evidence that students are deficient or “semilingual.” One conclusion about code switching that is relevant to the examination of mathematics learning in classrooms with bilingual students is that code switching should not be seen as primarily a reflection of language proficiency or the ability to recall (Valdes-Fallis, 1978).

I take a view of discourse as more than sequential speech or writing, using Gee’s (1996) definition of *Discourse*:

A Discourse is a socially accepted association among ways of using language, other symbolic expressions, and “artifacts,” of thinking, feeling, believing, valuing and acting that can be used to identify oneself as a member of a socially meaningful group or “social network,” or to signal (that one is playing) a socially meaningful role. (p. 131)

Next, I highlight some distinctions between the notions of register, discourse, and Gee’s (1996) definition of *Discourses*. Gee’s definition is not the usual one used in linguistics textbooks, which define *discourse* as “a sequence of sentences that ‘go together’ to constitute a unity, as in conversation, newspaper columns, stories, personal letters, and radio interviews” (Finegan & Besnier, 1989, p. 526). According to Gee’s definition, Discourses are more than sequential speech or writing and involve more than the use of technical language; they also involve points of view, communities, and values. Mathematical Discourses (in Gee’s sense) include not only ways of talking, acting, interacting, thinking, believing, reading, and writing but also mathematical values, beliefs, and points of view of a situation.

Gee (1999) also discussed the meaning of words:

A situated view of the meaning of words means that the meanings of words are not stable and general. Rather words have multiple and ever changing meanings created for and adapted to specific contexts of use. At the same time, the meanings of words are integrally linked to social and cultural groups in ways that transcends individuals. (p. 40)

With this view of word meaning, if vocabulary or registers are seen as stable and general or are defined as individual phenomena, a situated-sociocultural perspective is not compatible with either the acquiring vocabulary or the constructing multiple meaning perspectives. Although register markers include vocabulary, phonol-

ogy, morphology, syntax, and semantics, in mathematics education we may have focused mainly on the semantic aspects of register. Although the notion of register can be interpreted to include nonlinguistic behavior, such as interactional patterns and body language, the notion of Discourses as defined by Gee explicitly highlights the use of gestures and raises the use of artifacts. Although register may be an inherently social concept, Gee's definition of Discourse reminds (and perhaps forces) us to include more than words and meanings. Gee (1999) emphasized that "Discourses always involve more than language" (p. 25) and that aspects other than language, such as interactional and nonlanguage symbol systems, should be included in discourse analysis. Gee's definition of Discourses directs us to consider the importance of gestures, artifacts, practices, beliefs, values, and communities in mathematical communication.

Participating in classroom mathematical Discourse practices can be understood in general as talking and acting in the ways that mathematically competent people talk and act. These practices involve much more than the use of technical language. Gee (1996) used the example of a biker bar to illustrate the ways that any Discourse involves more than technical language. To look and act like one belongs in a biker bar, one has to learn much more than a vocabulary. Although knowing the names of motorcycle parts, makes, and models may be helpful, it is clearly not enough. In the same way, knowing a list of technical mathematical terms is not sufficient for participating in mathematical Discourse.

There is no one mathematical Discourse or practice (for a discussion of multiple mathematical Discourses, see Moschkovich, 2002). Mathematical Discourses involve different communities (mathematicians, teachers, or students) and different genres (explanations, proofs, or presentations). Practices vary across communities of research mathematicians, traditional classrooms, and reformed classrooms. However, even within each community, there are practices that count as participation in competent mathematical Discourse. As Forman (1996) pointed out, particular modes of argument, such as precision, brevity, and logical coherence, are valued. In general, being precise, explicit, brief, and logical and abstracting, generalizing, and searching for certainty are highly valued activities in mathematical communities. For example, claims are applicable only to a precisely and explicitly defined set of situations, as in the statement "Multiplication makes a number bigger, except when multiplying by a number smaller than 1." Many times, claims are also tied to mathematical representations, such as graphs, tables, or diagrams. Generalizing is also a valued practice, as in the statements "The angles of any triangle add up to 180 degrees," "Parallel lines never meet," or " $a + b$  will always equal  $b + a$ ." Imagining (e.g., infinity or zero), visualizing, hypothesizing, and predicting are also valued Discourse practices.

A situated-sociocultural perspective focusing on participation in mathematical Discourse practices can serve to broaden the analytical lens, complicate our view of language, and generate different questions. In the next section, I use the follow-

ing questions, selectively and loosely following Gee's (1999) questions for Discourse analysis, to examine two mathematical discussions:

1. *Situated meanings*: What are the situated meanings of some of the words and phrases that seem important in the situation?

2. *Resources*: What are the multiple resources students use to communicate mathematically? What sign systems are relevant in the situation (speech, writing, images, and gestures)? In particular, how is "stuff" other than language relevant?

3. *Discourses*: What Discourses are involved? What Discourses are being produced in this situation? What Discourses are relevant (or irrelevant)? What systems of knowledge and ways of knowing are relevant (and irrelevant) in the situation? How are they made relevant (and irrelevant) and in what ways? What connections are made to Discourses outside the immediate situation? In particular, what Discourse practices are students participating in that are relevant in mathematically educated communities or that reflect mathematical competence?

### MATHEMATICAL DISCUSSIONS IN CLASSROOMS WITH BILINGUAL STUDENTS

The two examples presented next are used to show the complexity that the use of a situated and sociocultural perspective as an analytical lens brings to the study of bilingual mathematical discussions. The first example shows how the vocabulary perspective can fail to capture students' competencies in communicating mathematically. The second example shows that the multiple-meanings perspective is not sufficient for describing all of the resources that students use.

#### Example 1: Describing a Pattern

The first example is from a classroom of sixth-grade through eighth-grade students in a summer mathematics course. The students constructed rectangles with the same area but different perimeters and looked for a pattern to relate the dimensions and the perimeter of their rectangles. Following is a problem similar to the one they were working on:

1. Look for all the rectangles with area 36 and write down the dimensions.
2. Calculate the perimeter for each rectangle.
3. Describe a pattern relating the perimeter and the dimensions.

In this classroom, there was one bilingual teacher and one monolingual teacher. A group of four students were videotaped as they talked in their small group and with the bilingual teacher (mostly in Spanish). As they attempted to describe the pattern in their group, they searched for the word for *rectangle* in Spanish. The stu-

dents produced several suggestions, including *ángulo* (“angle”), *triángulo* (“triangle”), *rángulos*, and *rangulos*. Although these students attempted to find a term to refer to the rectangles, neither the teacher nor the other students provided the correct word, *rectángulo* (“rectangle”), in Spanish.

Later, a second teacher (monolingual English speaker) asked several questions from the front of the class. One of the students in this small group, Alicia, tried to describe a relation between the length of the sides of a rectangle and its perimeter (see Figures 1 and 2):

Teacher B: [Speaking from the front of the class] Somebody describe what they saw as a comparison between what the picture looked like and what the perimeter was.

Alicia: The longer the, ah ... the longer [traces the shape of a long rectangle with her hands several times] the, ah ... the longer the *rángulo* [angle], you know the more the perimeter, the higher the perimeter is.

An analysis of this excerpt using the vocabulary perspective would focus on this student’s failed attempt to use the right word, *rectangle*. Focusing on how missing vocabulary was an obstacle would not do justice to how this student successfully communicated a mathematical description. If we were to focus only on Alicia’s inaccurate use of the term *rángulo*,<sup>2</sup> we might miss how she used resources from the situation and how her statement reflected practices valued in mathematical Discourse. Using the vocabulary perspective to analyze this student’s attempt (or failure) to use the right word would disregard her use of situational resources to communicate mathematically. If we move from a focus on the right word, we can begin to see this student’s competence. Alicia’s competence only becomes visible if we use a perspective of communicating mathematically that includes gestures and objects as resources.

A situated–sociocultural perspective allows us to consider the nonlanguage resources from the situation that the student used. Alicia used gestures to illustrate what she meant, and she referred to the concrete objects in front of her, the drawings of rectangles, to clarify her description. Alicia also used her native language as a resource. She interjected an invented Spanish word into her statement. In this way, a gesture, objects in the situation, and the student’s first language served as resources for describing a pattern. Even though the word that she used for rectangle does not exist in either Spanish or English, it is very clear from looking at the situation that Alicia was referring to a rectangle. It is also clear from her gestures that even though she did not mention the words *length* or

<sup>2</sup>Although the word does not exist in Spanish, it might be best translated as “rangle,” perhaps a shortening of the word *rectángulo*.



FIGURE 1 Alicia describing a rectangle: Part 1.



FIGURE 2 Alicia describing a rectangle: Part 2.

*width*, she was referring to the length of the side of a rectangle that was parallel to the floor.

Using a situated–sociocultural perspective, we can also ask what mathematical Discourse practices are relevant to this situation. Describing patterns is considered a paradigmatic practice in mathematics, so much so that mathematics is often de-

defined as “the science of patterns” (Devlin, 1998, p. 3). Alicia certainly described a pattern correctly. The rectangle with area 36 that has the greatest perimeter (74) is the rectangle with the longest possible length, 36, and shortest possible width, 1. As the length gets longer, say in the comparison of a rectangle of length 12, width 3, and perimeter 30 with a rectangle of perimeter 74, the perimeter does in fact become greater. Although Alicia was missing crucial vocabulary, she did appropriately (in the right place, at the right time, and in the right way) use a construction commonly used in mathematical communities to describe patterns, make comparisons, and describe direct variation: “The longer the \_\_\_\_\_, the more (higher) the \_\_\_\_\_.”

A situated–sociocultural perspective opens the way for seeing complexity and competence. Analyzed from this perspective, this example, instead of highlighting only the obstacles this student faced, points to the way the student used resources from the situation to communicate mathematically. Including not only vocabulary but also the gestures and objects provided the tools for describing the details of what this competence entailed. Making a connection to mathematical Discourse practices also widened what counts as competence.

Different implications for instruction follow from the vocabulary and situated–sociocultural perspectives. Certainly, Alicia needs to learn the word for rectangle, ideally in both English and Spanish, but instruction should not stop there. Rather than only providing the correction of her use of *rángulo* or the recommendation that she learn vocabulary, instruction should also build on Alicia’s use of gestures, objects, and description of a pattern.

### Example 2: Clarifying a Description

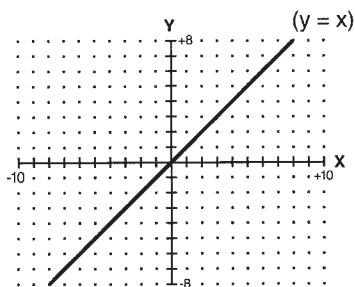
Although the first example fits the expectation that bilingual students need to acquire vocabulary, the vocabulary perspective was not sufficient to describe the student’s competence. The second example highlights the limitations of the vocabulary perspective for describing mathematical communication when students are not missing vocabulary. In the following discussion, the two students were not struggling with missing vocabulary in either Spanish or English. Instead, they used both languages for a purpose not related to vocabulary, clarifying the mathematical meaning of a description. The vocabulary perspective seems particularly limited for analyzing such cases.

The second example was taken from an interview after school. These two ninth-grade students had been in mainstream English-only mathematics classrooms for several years. One student in this example, Marcela, had some previous mathematics instruction in Spanish. These two students were working on the problem shown in Figure 3.

They had graphed the line  $y = -0.6x$  on paper and were discussing whether this line was steeper or less steep than the line  $y = x$  (see Figure 4).



8a. If you change the equation  $y=x$  to  $y=-0.6x$ , how would the line change?



A. The steepness would change.  
Why or why not?

___ NO	___ YES	___ STEEPER
		___ LESS STEEP

FIGURE 3 Problem for Example 2.

Giselda first proposed that the line was steeper, then less steep. Marcela had repeatedly asked Giselda if she was sure. After Marcela proposed that the line was less steep, she proceeded to explain this choice to Giselda:

Marcela: No, it's less steeper.

Giselda: Why?

Marcela: See, it's closer to the  $x$  axis [looks at Giselda]. ... Isn't it?

Giselda: Oh, so if it's right here ... it's steeper, right?

Marcela: Porque fíjate, digamos que este es el suelo, entonces, si se acerca más, pues es menos steep. ... 'Cause see this one [referring to the line  $y = x$ ] ... is ... está entre el medio de la  $x$  y de la  $y$ . Right? [Because look, let's say that this is the ground, then, if it gets closer, then it's less steep. ... 'Cause see this one is between the  $x$  and the  $y$ . Right?]

Giselda: [Nods in agreement.]

Marcela: This one [referring to the line  $y = -0.6x$ ] is closer to the  $x$  than to the  $y$ , so this one [referring to the line  $y = -0.6x$ ] is less steep.

The vocabulary perspective is not very useful for analyzing this example. Marcela, rather than struggling with vocabulary or using Spanish to fill in for a missing English word, used her first language to clarify a mathematical description. The following analysis shows how Marcela's competence involved more than knowing the meanings of *steeper* and *less steep*.

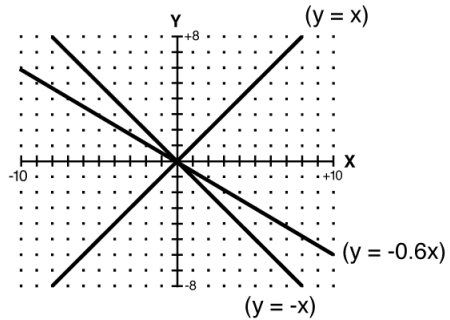


FIGURE 4 Lines drawn by Marcela and Giselda.

If we use a multiple-meanings perspective, we can begin to see that in this discussion, the two students were clarifying and negotiating the meanings of *steeper* and *less steep*. We could say that Marcela used the mathematics register as a resource to communicate mathematically. She used two constructions common in the school mathematics register: “Let’s say this is \_\_\_\_\_” and “if \_\_\_\_\_, then \_\_\_\_\_.”

However, the multiple-meanings perspective is not sufficient for describing Marcela’s competence. Using a situated–sociocultural perspective, we can ask, which nonlanguage resources from the situation did she use? This student used not only mathematical artifacts—the graph, the line  $y = x$ , and the axes—but also everyday experiences as resources.

The premise that meanings from everyday experiences are obstacles for communicating mathematically does not hold for this example. In fact, Marcela used her everyday experiences and the metaphor that the  $x$  axis is the ground (“Porque fíjate, digamos que este es el suelo” [“Because look, let’s say that this is the ground”]) as resources for explaining her description. Rather than sorting out multiple meanings between two registers, she used an everyday situation to clarify her explanation.

Using a situated–sociocultural perspective, we can also ask, what aspects of mathematical Discourse practices are relevant to this situation? Marcela’s explanations echoed mathematical Discourse practices that go beyond the use of constructions from the mathematics register. First, Marcela explicitly stated an assumption, a discursive practice valued in mathematical Discourse, when she said, “Porque fíjate, digamos que este es el suelo” (“Because look, let’s say that this is the ground”). Second, she supported her claim by making a connection to mathematical representations, another valued discursive practice in mathematical Discourse. She used the graph, in particular the line  $y = x$  and the axes, as references to support her claim about the steepness of the line. A situated–sociocultural perspective can help us to see that Marcela was participating in two discursive practices that reflect important values, stating assumptions explicitly and connecting claims to mathematical representations.

## CONCLUSIONS

This area of study brings together different views of mathematics learning and of language. Work in mathematics education provides us with several ways to think about what it means to learn mathematics. Learning mathematics can be seen as learning to carry out procedures and solve traditional word problems, constructing meanings, or participating in mathematical discourse practices. Work on bilingual mathematics learners needs to be informed by current views of learning mathematics as sense making (Lampert, 1990; Schoenfeld, 1992), developing sociomathematical norms (Cobb et al., 1993), and learning to participate in mathematical Discourse practices (Brenner, 1994; Brown et al., 1989, Forman, 1996; Forman, McCormick, & Donato, 1998; Greeno, 1994; Lave & Wenger, 1991). There are also different views of language as vocabulary, registers, and Discourses. The study of mathematics learning in classrooms with bilingual students also needs to be informed by current perspectives on communication in classrooms (Ballenger, 1997; Cazden, 1986, 1993; Heath, 1983; Mehan, 1979) and bilingualism (Hakuta & Cancino, 1977; Valdes-Fallis, 1978; Zentella, 1997).

A perspective of learning mathematics as acquiring vocabulary has been used to describe how students learn to solve English word problems and understand mathematical texts. A perspective of learning mathematics as constructing multiple meanings across registers has uncovered possible misunderstandings in classroom conversations. This second perspective has been useful in pointing out ways to support English-language learners in communicating mathematically: clarifying multiple meanings, addressing the conflicts between two languages explicitly, and discussing the different meanings students may associate with mathematical terms in each language.

However, these two perspectives have limitations. Seeing learning mathematics as acquiring vocabulary is not sufficient for describing different types of mathematical discussions, situational resources, or student competence. Focusing on the obstacles between the everyday and the mathematics register can obscure how everyday meanings can be resources for mathematical discussions. Both of these perspectives can be interpreted as reducing mathematical discourse to the use of vocabulary or presenting a deficiency model of bilingual students as mathematics learners. An accurate description of mathematical communication for bilingual students needs to include not only an analysis of the difficulties but also the multiple resources students use to communicate mathematically. A situated-sociocultural perspective can broaden the analytical lens and generate different questions, such as a consideration of the situational resources students use and the ways that mathematical Discourses are relevant to a situation.

The first example of a bilingual mathematical discussion showed that even when students are missing a word, students' first language and aspects of the situation, such as gestures and objects, can be resources for communicating mathemati-

cally. The second example showed that the everyday register and a student's first language can be resources rather than obstacles for learning mathematics. Although the first example showed students who recognized a difficulty in finding the right vocabulary term, not all bilingual mathematical discussions serve this purpose. The students in the second example used both languages to explain a problem solution rather than finding or translating a word.

The previous examples illustrated several aspects of how bilingual students communicate mathematically that only become visible when a situated–socio-cultural perspective is used:

- Learning to participate in mathematical Discourse is not merely or primarily a matter of learning vocabulary. During conversations in the mathematics classroom, students are also learning to participate in valued mathematical Discourse practices, such as being precise or using representations to support claims.
- Some of the resources bilingual students use to communicate mathematically are gestures, objects, everyday experiences, their first language, code switching, and mathematical representations.
- There are multiple uses of Spanish in mathematical conversations between bilingual students. Some students use Spanish to label objects. Other students use Spanish to explain a concept, justify an answer, or describe a mathematical situation.
- Bilingual students bring varied competencies to the classroom. For example, even a student who is missing vocabulary may be proficient in using mathematical constructions or presenting clear arguments.

The situated–sociocultural perspective used to examine these two mathematical discussions can expand and complicate our view of how bilingual students learn mathematics. Even when students are learning to communicate mathematically in their second language, they are doing much more than finding the right word or struggling with multiple meanings (Ballenger, 1997). As students participate in mathematical discussions, they are using resources such as their first language, gestures, and objects. They are also participating in Discourse practices that reflect the values of the discipline, such as being explicit about assumptions, connecting claims to representations, imagining, hypothesizing, and predicting. If we use a situated–sociocultural definition of Discourse, we can widen what counts as competence. To do this, we should maintain two central assumptions: Discourses are more than language; and meanings are multiple, changing, situated, and sociocultural.

### Implications for Instruction

Any perspective that focuses on the obstacles bilingual students face can easily become a deficiency model (Garcia & Gonzalez, 1995; González, 1995) of bilingual students as mathematics learners. Descriptions of mathematical discussions in

classrooms with bilingual students need to consider not only the obstacles that students face but also the resources students use to communicate mathematically. A situated–sociocultural perspective points to several aspects of classroom instruction that need to be considered. Classroom instruction should support bilingual students' engagement in conversations about mathematics that go beyond the translation of vocabulary and involve students in communicating about mathematical concepts.

A situated–sociocultural perspective on learning mathematics can help to shift the focus of mathematics instruction for English-language learners from language development to mathematical content. The two examples presented show that English-language learners can and do participate in discussions where they grapple with important mathematical content, even if they do not always use the right words and even if they switch from English to Spanish. One of the goals of mathematics instruction for bilingual students should be to support all students, regardless of their proficiency in English, in participating in discussions about mathematical ideas. Teachers can move toward this goal by providing opportunities for bilingual students to participate in mathematical discussions and by learning to recognize the resources that bilingual students use to express mathematical ideas.

Classroom conversations that include the use of gestures, concrete objects, and the student's first language as legitimate resources can support students in learning to communicate mathematically. Instruction needs to support students' use of resources from the situation or the everyday register, in whichever language students choose. Lastly, assessments of how well students communicate mathematically need to consider more than their use of vocabulary. These assessments should include how students use the situation, the everyday register, and their first language as resources as well as how they make comparisons, explain conclusions, specify claims, and use mathematical representations.

Understanding the mathematical aspects of what students say and do can be difficult when teaching, perhaps especially when working with students who are learning English. It may not be easy (or even possible) to sort out which aspects of a student's utterance are the result of the student's conceptual understanding or of a student's English-language proficiency. However, if the goal is to support student participation in mathematical discussions, determining the origin of an error is not as important as listening to the students and uncovering the mathematical competence in what they are saying and doing. It is only possible to uncover students' mathematical competence if we use a complex perspective of what it means to communicate mathematically.

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