ZDM Mathematics Education (2010) 42:91–104
DOI 10.1007/s11858-009-0213-5

ORIGINAL ARTICLE

# A social perspective on technology-enhanced mathematical learning: from collaboration to performance

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Accepted: 24 August 2009 / Published online: 15 September 2009


**Abstract** This paper documents both developments in the technologies used to promote learning mathematics and the influence on research of social theories of learning, through reference to the activities of the International Commission on Mathematical Instruction (ICMI), and argues that these changes provide opportunity for the reconceptualization of our understanding of mathematical learning. Firstly, changes in technology are traced from discipline-specific computer-based software through to Web 2.0-based learning tools. Secondly, the increasing influence of social theories of learning on mathematics education research is reviewed by examining the prevalence of papers and presentations, which acknowledge the role of social interaction in learning, at ICMI conferences over the past 20 years. Finally, it is argued that the confluence of these developments means that it is necessary to re-examine what it means to learn and do mathematics and proposes that it is now possible to view learning mathematics as an activity that is *performed* rather than passively acquired.

**Keywords** Technology · Collaboration · Performance · Learning · Mathematics

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## 1 Introduction

In this paper, we discuss three major shifts in technology in mathematics education, in the context of the last two decades of the history of the International Commission on Mathematical Instruction (ICMI). In the first part of the paper, we address the shift from computer-based mathematics software to Web 1.0 and Web 2.0-based learning tools and environments. In the second part of the paper, we focus on the emergence of social perspectives on the use of technology in mathematics education. Lastly, we explore the multimodal and collaborative affordances of Web 2.0 and identify a potential emergent shift toward social performance (as in the Arts) in mathematics teaching, learning and doing.

## 2 Computer based to Web based

We start this section by looking at the role of technology in mathematics education as depicted in the proceedings of two International Congresses on Mathematical Education (ICME): ICME-6 (Hirst and Hirst 1988) and ICME-7 (Gaulin et al. 1994). We selected ICME-6 and ICME-7 as a starting point, as their time period represents a turning point (as we will discuss below) toward a much more increased focus on the role of technology in mathematics education. In the latter part of this section, we compare and contrast the role of technology in mathematics education, as represented in ICME-6 and ICME-7, with today's situation, as in the proceedings of ICME-10 (Niss 2008), proceedings of the 17th ICMI study (Hoyles et al. 2006) and the associated study volume (Hoyles and Lagrange 2009), and recent research projects on teaching with technology.

## 2.1 ICME-6 and ICME-7

Shumway (1989), reflecting on ICME-6, observes that when the computer was initially used in education (starting around 1965), it was valued as a tool for computation and programming. Later, it was valued as a tool for drill and practice, teacher utility, information management and tutorial uses. He notes that by 1988, the focus seemed to shift to computations, graphics, simulations, concept learning and problem-solving. Fey (1993), reflecting on ICME-7, adds that Shumway's summary also described the situation of ICME-7.

> In the miniconference, working groups, and special lectures there was very little evidence of interest in computer programming as a vehicle for learning mathematics, in computer tutors that control the learning environment as electronic instructors in drill-and-practice software, or in the variety of ways that computers can be used to assist with information management tasks of teaching. The strongest theme in most sessions was the search for powerful computer and calculator tools that would enable students to conduct mathematical investigations that solve important real problems and yield understanding of important concepts. (p. 7)

The emergence of a view of technology as enabling mathematical investigation and facilitating the development of student mathematical thinking and understanding was an important turning point in the history of technology in mathematics education.

There is strong evidence in the proceedings of ICME-6 (Hirst and Hirst, 1988) of the emerging focus of mathematics educators on technology. Many of the study groups, although not explicitly interested in technology, did address how technology (calculators and microcomputers) might affect their particular focus. A sampling of the ideas discussed at the conference includes the following:

1. Technology allows students and teachers to engage with more complex and realistic applications, in a less laborious way, and offers new possibilities of modeling, exploration and simulation.
2. Calculators and microcomputers do not replace the teacher.
3. The graphic capability of new technology helps support the teaching of geometry.
4. Changes in curriculum are just beginning, with questions about obsolete, as well as new, math skills emerging due to the nature of the technology.
5. The use of technology might help create a new style of mathematical thinking as well as pedagogy, with a focus on "what-if".
6. Technology is changing quickly and it is difficult for research to keep pace with it.

ICME-7 was much more representative than ICME-6 of the turning point, in our view of technology in mathematics education, because of its explicit focus on technology in mathematics education. ICME-7 was the first ICME to focus intensely on technology in mathematics education. For example:

- The first day of ICME-7 offered a 3.5 h miniconference on calculators and computers. The participants were invited to join one of five groups: students, ages 5–11; students, ages 11–16; students, ages 15–18; mathematics undergraduate students; teacher education students. As Fey (1993) notes, "The miniconference made technology a prominent feature of the ICME-7 right from the start, and this attention was continued throughout the week-long Congress program." (p. 6). Fey also notes that "Furthermore, it is evident from the program for other working groups that sessions of those groups devoted considerable attention to the impact of technology in all aspects of mathematics education" (p. 7).
- There were three working groups on technology, which met for four 1.5-h sessions each: "Impact of Calculators on Elementary School Curricula", "Technology in Service of the Mathematics Curriculum" and "TV in the Mathematics Classroom".
- The Congress included important lectures by Celia Hoyles, Benoit Mandelbrot and Seymour Papert. Mandelbrot (1994) noted in his talk that "fractals—together with chaos, easy graphics, and the computer—enchant many young people, which, in turn, makes them excited about learning mathematics." (p. 77). He also discussed how the computer can be used to model and experiment with mathematical ideas: "You program this silly little formula into your trusty personal computer or workstation, and suddenly everything breaks loose." (p. 85).
- There were numerous software presentations at the poster displays and exhibits.

Lichtenberg (1993), reporting on the ICME-7 working group dealing with the impact of the calculator, notes that "Common threads among the papers in this working group include encouraging mathematical thinking, exploration, mental work, having fun with mathematics, and posing questions like "What do you think will happen if …?""(p. 19). Fey (1993) notes in his reflective paper on ICME-7 that "the most frequently mentioned software was geometry drawing tools, especially the Geometric Supposer series, *Cabri-geometre*, and the *Geometer's Sketchpad*. From people who had experienced the software, I heard consistent reports that promised benefits in teaching and learning" (p. 8). Fey also notes that "much of the most exciting development work applying calculators, computers, and video to mathematics education is occurring in North America." Also persisting was the idea of immersive mathematical microworlds (pioneered by Seymour Papert in the late 1960s). However, technology in the classroom was not a worldwide phenomenon. Mayo (1993) notes stories from delegates of Nigeria, the Dominican Republic, India and China where teachers not only lacked computers, but also such things as chairs, desks and other classroom resources that North Americans take for granted.

ICME-7 also included signs of the emergent focus on online modes of communication in mathematics education. One of the working groups in ICME-7 focused on mathematics in distance learning contexts. This working group discussed a variety of communication methods, including written material, material on CD-ROM, the use of fax, one-way video with two-way audio, as well as the more expensive and complex two-way video with two-way audio. A few cases were also shared where email was used as a method of communication. "Though currently limited to the transmission of text, developments which allow the transmission of graphics, sound, and even slow-scan video are beginning." (Knight 1994, p. 213)

## 2.2 Current situation

What has changed since the time of ICME-6 and ICME-7? Through a review of the proceedings of ICME-10 (Niss 2008) and the 17th ICMI study (Hoyles et al. 2006) and the associated ICMI 17 study volume (Hoyles and Lagrange 2009), as well as our own experience with mathematics education technology, we can identify three shifts in focus: from computer-based applications to Web-based learning; from Web 1.0 to Web 2.0; and from thinking about technology to thinking *with* technology.

### *2.2.1 From computer based to the Web based*

A word search of the 557-page proceedings of the 17th ICMI study (Hoyles et al. 2006) reveals that words referring to Web-based aspects of mathematics (such as "online", "Internet", and "Web-based") appear 260 times. A similar search of the 559-page proceedings of ICME-10 (Niss 2008) results in 235 occurrences. There are also references to online games, applets, online textbooks and mathematical online communication and collaboration (see, e.g., Drijvers and Stacey 2008; Healy and Kaput 2008; Afanasiev and Crowe 2008; Beatty and Moss 2006; Jarvis 2006; Sloan and Olive 2006).

In the last 20 years, the present authors have worked in mathematics education at the elementary and secondary school levels, as teachers, district mathematics consultants, software designers and developers, and mathematics education professors. During this time, our focus has shifted from computer-based software to Web-based learning objects and the design of online mathematics learning experiences. Before 1990 and continuing until about 2000, we used and trained other teachers to use graphic calculators and a variety of computer-based software that offer students opportunities for exploring mathematical concepts and relationships in a non-scripted fashion (such as, the Geometric Supposer, Geometer's SketchPad and spreadsheets). However, in the late 1990s, our focus on computer-based software was overtaken by our interest in the potential of Web-based environments, which became an umbrella for a variety of mathematics experiences with technology in mathematics education.

### *2.2.2 From Web 1.0 to Web 2.0*

Our digital age has been labeled as an information revolution (as contrasted with the industrial revolution). Schrage (2001) suggests that this label misses the essence of the paradigm shift.

> *In reality, viewing these technologies through the lens of "information" is dangerously myopic*. The value of the Internet and the ever-expanding World Wide Web does not live mostly in bits and bytes and bandwidth. To say that the Internet is about "information" is a bit like saying that "cooking" is about oven temperatures; it's technically accurate but fundamentally untrue. (p. 1; original emphasis)

Schrage argues that a more appropriate label is *relationship revolution*.

> *The so-called "information revolution" itself is actually, and more accurately, a "relationship revolution."* Anyone trying to get a handle on the dazzling technologies of today and the impact they'll have tomorrow, would be well advised to re-orient their worldview around relationships…. *When it comes to the impact of new media, the importance of information is subordinate to the importance of community*. The real value of a medium lies less in the information that it carries than in the communities it creates. (pp. 1–2; original emphasis)

Lankshear and Knobel (2006) suggest that the relatively recent "development and mass uptake of digital electronic technologies" represent changes on a "historical scale", which "have been accompanied by the emergence of different (new) ways of thinking about the world and responding to it." (pp. 29–30). These new ways of thinking can be characterized as more "participatory", "collaborative" and "distributed", and less "published", "individuated" and "author-centric"… also less "expert-dominated." (Lankshear and Knobel, 2007, p. 9).

The Web 2.0 paradigm views a Website not as a static read-only page, but as a dynamic read/write environment (such as a wiki) where users interact and co-generate content and experiences. In the last few years, the present authors have been focusing on Web 2.0 affordances, especially (1) collaborative knowledge-building environments (such as wikis) that entrust users as co-authors or co-developers and potentially tap into their collective intelligence and (2) the increasing support for multimodal communication. This shift is reflected in a number of current projects as discussed below.

*2.2.2.1 Read/write learning objects* Gadanidis et al. (2007) are developing learning objects that allow users to annotate a given state of a learning object and to share the state and the annotations with others (as a URL that is sent in an e-mail or posted on a Website). The annotations currently are text only; however, they are in the process of developing video annotations (captured by the learning object using a webcam) and drawing annotations (using a drawing tool built into the learning object). The ability to edit and annotate a learning object creates the potential for changing the relationships between students and learning objects. The opportunity to share "versions" of a learning object with others also creates the potential for changing the relationships among students. This is a view supported by Afanasiev and Crowe (2008) during ICME-10, who suggest that "a student can learn by solving a geometrical problem being driven by a remote tutor, who manages the same pictorial constructions" (p. 535).

*2.2.2.2 The affordances of wikis* Gadanidis et al. (2008) note that the idea of a wiki has 'stained' their thinking about almost everything they do in development and research projects and in their teaching. In their online teaching, they can no longer imagine using a discussion tool that is not wiki-based. In 2004, Gadanidis designed an online discussion platform called idea construction zone (ICZ), which allowed for wiki postings (postings that can be edited by others in the discussion), synthesis postings (where a number of postings can be selected and their content merged into a single posting that the user can edit, with all authors credited), embedded drawings within postings using a built-in drawing tool (see example in Fig. 1), embedded video (or audio) captured using a webcam within a posting, embedding other multimedia within postings (such as JPEG, animated GIF, and Flash swf) or hyperlinking to external resources. Since 2005, ICZ has been used to teach graduate, preservice and continuing teacher education courses. It has also been used in research projects between Canada and Brazil and Canada and Tanzania, involving students, teachers and researchers.


**Fig. 1** An elementary preservice teacher uses the Draw Tool to show three representations of 'parallel' lines

Bold (2006) suggests that "the wiki is quickly understood as an alternative discussion tool […] use of the wiki meets little resistance and only temporary consternation at how it works" (p. 11). However, Gadanidis et al. (2008), Borba and Gadanidis (2008) and Grant (2006) note that they have experienced a resistance on the part of students in using the read/write features of a wiki. Students experience difficulty in editing the work of others. There are a number of possible reasons for this. Firstly, as a wiki text is communally edited, it is difficult to distinguish authorship of ideas. Richardson (2006) notes that "[p]hilosophically, wikis can play havoc with the traditional ideas of copyright and intellectual property" (p. 63). Participating in a wiki may make some students feel that they have given up ownership of their ideas. However, as Landow (2006) notes, "most of our intellectual endeavors involve collaboration, but we do not always recognize that fact" (p. 138). For example, the present paper was read by a number of anonymous reviewers who offered critique and suggestions for improvement, and we used their feedback to improve our paper, without giving them credit for their contributions. Secondly, the dominant school experience is that students write their school work in private and that typically confidential (rather than public) suggestions for improvement come from the instructor. Lankshear and Knobel (2006) note that "What seems to be happening is that the day-to-day business of school is still dominated by conventional literacies, and engagement with 'new' literacies is largely confined to learners' lives in spaces outside of schools and other formal educational settings." (p. 30). Thirdly, using a wiki in an educational setting is a new and different experience for teachers, and they need to develop new ways of looking at what it means to teach and learn in collaborative knowledge construction environments such as wikis. As Lankshear and Knobel note, "We are presently at a point in the historical-cultural development of literacy where we don't really know how to deal educationally with these new literacies" (p. 30). However, we need to be cautious with respect to the new literacies that we attribute to students, as a result of their out-of-school digital experiences. For example, the typical social networking experiences that are becoming prominent in students' out-of-school lives do not necessarily involve their editing the ideas of others. For instance, the type of collaborative knowledge construction that is evident in Wikipedia is not typically part of the social networking experience of students when using such popular environments as Facebook or Youtube.

The widespread use of wikis is fairly recent, and they are not mentioned in the proceedings of ICME-10 (Niss 2008) and 17th ICMI study (Hoyles et al. 2006). However, research on collaborative affordances of technology was not absent in these studies. For example, the research of Beatty and Moss (2006), working with fourth-grade students in Knowledge Forum (an online environment with collaborative affordances), indicates that Knowledge Forum supported "collaborative problem-solving" and "supported students in developing a community practice of offering evidence and justification for their conjectures" (Beatty and Moss 2006, p. 27). Also Dor Abrahamson and Uri Wilensky presented a paper at ICME-10 that "explored the use of a rich, collaborative and interactive computer-based learning environment" (Li and Wisenbaker 2008).

### *2.2.3 From thinking about technology to thinking with technology*

Healy and Kaput (2008) (in a paper/plenary delivered at ICME10) note that "More recently, it is being recognized that greater emphasis should be placed on the need to understand the mathematical practices that emerge in complex, self-organizing, interacting systems, involving multiple learners and teachers using technological tools within and across a variety of settings" (p. 356). When we immerse ourselves in using a technology (and this immersion is a critical component), we naturally think *with* that technology, whether it is the technology of the printed text, or the technology of the word processor we are using to author this paper, or the technology of a wiki we use to design and teach an online course on mathematics for teachers. Levy (1997) suggests that technology is itself an actor in the collaborative process, and not simply a tool used for human intentions. When we immerse ourselves in using a technology, either individually or in collaboration with others, the technology becomes an integral component of the cognitive ecology that is formed. Borba and Villarreal (2005) add that humans-with-media form a collective where new media also serve to disrupt and reorganize human thinking. Borba (2009), examining possible future scenarios of Internet use in the mathematics classroom, suggests that "the Internet may "deconstruct" the structure of curriculum" (p. 457), as the Internet not only helps express ideas, but also "shape ideas and language" (p. 461). What has changed for us in the last few years is that the technology of the Web has become a pervasive environment, and its various affordances have become tools we increasingly and naturally think with. There is a qualitative difference here between immersion in a computer-based software like Geometer's SketchPad, which has a narrow application, and Web-based tool like a wiki, which has infused into various layers of our lives (teaching mathematics courses for teachers, teaching graduate courses, conducting international research where the wiki is used to bring together students for different countries, or creating a family wiki where pictures, videos and news are posted and discussed by family members overseas).

The mathematics education technology that we thought about in the ICME-7 era was computer based and mathematics specific. The mathematics education technology we think with today is Web based and it is less domain specific. Our thinking about mathematics education has been disrupted and reorganized as we use and think *with* the technology of the Web. For example, using a wiki in our online teaching is a very different experience than teaching the same groups of students in a physical classroom. It is also very different from using Web 1.0 tools such as WebCT. Using a wiki does not only disrupt and reorganize our thinking about how we organize classroom interaction: it also becomes a lens that changes how we see other aspects of online teaching, such as course content, evaluation practices, our role as instructors, and generally what constitutes knowledge and how it is or should be constructed in an online environment.

The Web's shift from text-based, read-only communication to multimodal, read/write communication is not simply a quantitative change: it is not just a case of having more communication modes. It is a qualitative change, analogous to the change that occurred when we moved from an oral to a print culture. However, our understanding of what this change implies for mathematics education (and education in general) is emergent and not fully conceptualized or articulated. In this context of increased social affordances of communication technologies, which are becoming commonplace in out-of-school settings, and are beginning to enter educational settings, it is important to consider what educational theories of social interaction might be developing alongside the technological changes. We address this issue in the first section that follows. Also, in the second section, we look to the future and speculate whether the social and multimodal affordances of new media, which introduce 'audience' and 'multimodal communication', might serve as a fertile ground for developing social theories of learning and teaching with technology along performative paradigms.

## 3 The emergence of social perspectives on using digital tools in learning mathematics

Schrage's (2001) observation that ICTs have instigated a *relationship revolution* rather than an *information revolution*, because of the communities that emerge through the use of digital tools, parallels Lerman's (2000) observation of an emergence of a social perspective on teaching and learning mathematics. At the 9th ICME, held in Makuhari, Japan, Lerman presented a plenary lecture to the congress entitled *The Socio-cultural Turn in Studying the Teaching and Learning of Mathematics* (Lerman 2000). In his address, he argued that there was growing interest in and increasing research support for theories of learning in which thinking, reasoning and meaning making originate in and/or are mediated by social activity.

> The term social turn in my title is intended to signal something different, however, namely the emergence into the mathematics education research community of theories that see meaning, thinking and reasoning as products of social activity. (Lerman 2000, p. 157)

Prominent among social theories of learning are those of socio-culturalism and social constructivism. While these theories are fundamentally different in terms of the role of social interaction in learning, thinking and intellectual development, both assign a central role to social activity in mathematical learning. A summary of key elements of these theories is presented below.

### 3.1 Social constructivism

Constructivist theories of intellectual development assign an active role to the learner in the construction of unique and personal knowledge through the development of symbolic representations that are used to interpret and interact with the world (Noddings 1990). Intellectual growth begins, from a constructivist perspective, when a learner encounters ideas, phenomena or practices that conflict with their view of the world. This state of cognitive conflict, or *disequilibrium*, was identified by Piaget as fundamental to the process of intellectual development. Learning occurs when an individual is able to resolve the conflict by rearranging their cognitive structures in such a way that the conflict is accommodated and assimilated into the individual's cognitive structure. While this process portrays learning as an individualistic enterprise that takes place "in one's own head", more recent developments within constructivist theory acknowledge the role of interaction between teachers and learners and between learners themselves in provoking the necessary state of *disequilibrium* and also in the resolution of cognitive conflict (see, e.g., Cobb and Bauersfeld 1995). Thus, while social interaction can be a source of disequilibrium, it can also be the means by which a cognitive conflict is resolved by mediating the development of new knowledge and meaning structures.

### 3.2 Socio-culturalism

By contrast, socio-cultural theories of learning are founded on a position that intellectual development originates in, and so is not just facilitated by, social interaction. Learning is a process of enculturation into the practices of a learning community. Enculturation into the community requires the appropriation of modes of reasoning, discourse and knowledge creation that are accepted by the discipline around which the community is based. Learning mathematics in such a community means a learner must participate in debate about new ideas and practices, offer critique of others ideas and defend their own propositions via explanations and justifications (Goos et al. 2000).

In addition, socio-culturalism acknowledges the role of both cultural tools, such as language representations and sign systems, and physical artefacts, such as calculators and computers, in mediating learning. The appropriation of tools into accepted modes of reasoning and discourse is also an important part of the enculturation process. Thus, a learner must acquire more than facility with such tools: they must also appropriate new modes of reasoning, argumentation and knowledge validation in which digital tools are seamlessly integrated.

It is not the purpose of this paper to argue in favor of either theoretical position; the descriptions presented above are included to provide a backdrop for a review of the emergence of social perspectives on mathematical learning through the history of ICMI. The review presented below is inclusive of all theoretical frameworks that assign an important role to social interaction in the learning and teaching of mathematics.

### 3.3 The development of a social perspective on using digital tools to enhance mathematics learning through the activities of ICMI

A growing body of research is developing around the idea that there is now great potential to think *with* technology. Pea (1985, 1993a, b), for example, draws on a Vygotskian view of intellectual development to argue that learning and reasoning should now be considered the activity of a system, which involves minds, social contexts and tools such as computers, that is, that thinking is *distributed* among and between these elements. It has been argued by authors such as Goos et al. (1999, 2000, 2003) and Geiger (2005, 2006) that productive social interaction in mathematics classrooms can be mediated by technology. Goos and associates developed a typology of technology use in which two metaphors, *Technology as Partner and Technology as Extension-of-self*, are used to describe patterns of student–student-technology behavior where the boundaries between human and technological agents are blurred when students learn and use mathematics. Consistent with this view, Borba and Villarreal (2005) propose a unit of analysis for research in mathematics learning in technology-rich environments, *humans-with-media*, which recognize that the integral role technology can play in the reorganization of mathematical thinking and that the contributions of humans and ICTs in promoting learning are not easily separated.

While these examples of research are representative of a body of knowledge that recognizes the role of both technology and social interaction in mathematical learning, they do not in themselves demonstrate a broad acceptance of this view. To establish the validity of Schrage's (2001) observation and Lerman's (2000) claim of a shift in interest toward a more social view of teaching and learning, we reviewed selected documents that record the activities of ICMI over a period of approximately 20 years. To benchmark interest in the role of social activity in technology-influenced mathematics teaching and learning, we chose to review the proceedings of ICME 5 (Carss 1986) and proceedings of the first ICMI study *The Influence of Computers and Informatics on Mathematics and its Teaching* (Churchhouse 1986). These two sources were selected because it was in the 1980s that computer technologies became widely available to mainstream schools. Judgment of the growth of interest in social perspectives on the potential of technology to enhance mathematics teaching and learning will be achieved by contrasting evidence from the two sources in 1986 with a review of relevant sections of the proceedings of ICME10 (Niss 2008), the most recent congress for which proceedings are available, and the proceedings of ICMI's 17th study *Digital Technologies and Mathematics Teaching and Learning: Rethinking the Terrain* (Hoyles et al. 2006) and the associated ICMI 17 study volume (Hoyles and Lagrange 2009).

### *3.3.1 Early interest in technology for enhancing mathematics learning*

The proceedings of the Fifth International Congress on Mathematics Education (Carss 1986) include the report of a theme group, *The Role of Technology*, and a record of a plenary debate between Hugh Burkhardt and Philip Davis, entitled *The Microcomputer: Miracle or Menace in Mathematics Education*. The theme group report includes a vignette related to a presentation by Rosemary Fraser in which she invited the audience to participate as "pupils" in a simulated classroom. By working with participants in this way and using only a single microcomputer and two programs, *Pirates* and *Eureka*, Fraser "illustrated clear role-shifting with children taking over computer and teacher roles, resulting naturally in problem-solving and open-ended activities" (Carss 1986, p. 160). This indicates a clear interest in pedagogical aspects of computer use in school mathematics classrooms and, in particular, on the role technology can play is transforming traditional classroom roles, although there is no indication of the type of social interaction which took place during this session. The theme group was also concerned with the potential of technology to transform mathematics itself, as well as the way it was taught and learned. This is illustrated by the following quote from the theme group's report.

> Computer science changes the philosophical foundations of mathematics. It affects what mathematics is to be taught, the development of symbolic systems, doing more experimental mathematics, new skills, observation, visualization, simulation and numerical verification. (Fraser et al. 1986, p. 174)

While the theme group report demonstrates an interest in both pedagogy as well as the way technology might change mathematics, specific discussion of pedagogy is limited to an acknowledgement that "Computer science changes pedagogical styles. The computer will alter the pupil teacher relationship" (p. 174). There is no discussion, however, of how the pupil teacher relationship will change or any comment on how peer interaction might be influenced by the availability of computer technology: a position which is understandable given that the use of digital technologies to enhance mathematics instruction was in its infancy and the most appropriate ways to take advantage of the benefits offered by technology were only just being developed.

The diagram that accompanies the quote above, Fig. 2, identifies the interactions that computers were perceived as having the power to influence.

While the range of different types of classroom interactions in the revised view of classroom activity has increased, the diagrams still suggest that pupils act as individual agents in the learning process and that negotiation between themselves and the teacher, while bidirectional, are conducted on a one-to-one basis. No space is allowed for student–student interaction or how this type of interaction might affect learning and teaching. This is unsurprising given the context in which computer

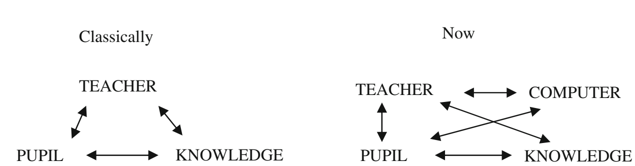


**Fig. 2** Pupil–teacher relationship (Fraser et al. 1986, p. 174)

technology of this time was designed, that is, only for interaction with individuals and not specifically to promote collaborative interaction.

At the same Congress, Burkhardt and Davis debated the potential benefits of computers in mathematics classrooms, with both outlining the advantages that were perceived to, at the time, lie within the capacities of the computer. Burkhart, in particular, painted a picture of a future classroom in which teachers would be supported by a digital teaching assistant, who would assume a substantial part of a teacher's load of explaining and managing task setting, thus freeing teachers to assist students with less directed activities such as problem-solving. There was no discussion of how technology could be used to facilitate productive interactions between teachers and students or between students and their peers. This also indicates that the use of technology to enhance collaborative classroom practices was not part of the thinking of that time.

In March of the same year, the first of ICMI's 18 studies was initiated via a symposium held in Strasbourg, Germany. The proceedings of that meeting, *The Influence of Computers and Informatics on Mathematics and its Teaching* (Churchhouse 1986), document the activity of the symposium under the following themes:

1. How do computers and informatics influence mathematical ideas, values and the advancement of mathematical science?
2. How can new curricula be designed to meet changing needs and possibilities?
3. How can the use of computers help the teaching of mathematics?

The report on the third theme opens with a discussion of what mathematics and mathematical activity might look like in a classroom of the future. The report argues that, in the future, the availability of computers in classrooms will mean:

> …the experimental aspects of mathematics assume greater prominence, and there is a corresponding wish to ensure that provision should be made for students to acquire skills in, and experience of, observing, exploring, forming insights and intuitions, making predictions, testing hypothesis, conducting trials, controlling variables, simulating, etc. (pp. 24–25)

This again indicates the developing belief that technology would change the way mathematics was taught and learned, although there is no commentary of how students might work together, or how students and teachers might work together during such activity. Later in this section, however, there is acknowledgement that computers have the potential to transform the roles of teachers, students and technology in the mathematics classrooms.

> …this creates new interactions and relationships between student, knowledge, computer and teacher. (p. 25)

While this statement highlights a developing sense that the nature of classroom interaction would change due to the introduction of computers, it is unclear what role was anticipated for technology in promoting social interaction between learners.

In summary, interest in computers during the late 1980s, in relation to the teaching of mathematics, focused principally on the capacity of digital technology to provide teachers with tools to assist the individual learner and in the way the mathematics itself, in schools, would be transformed. It was anticipated, however, that technology would play a role in changing the way teaching and learning took place in school classrooms including the ways in which students and teachers would interact, although the nature of such interaction was not yet clear.

### *3.3.2 Recent interest in technology and mathematics teaching and learning from a social perspective*

Moving forward to ICME-10, held in Copenhagen in 2004, the most recent Congress for which proceedings are available, there is a marked change in the attention given to social interaction in the process of learning and in the role of technology in mediating collaborative learning in mathematics. Celia Hoyles, in a plenary that described her work over two decades, reports:

> In a later project, WebLabs (http://www.weblabs.eu.com/), we extended our design work still further and shifted its focus to iteratively building tools and activity sequences in which students, in different sites across Europe, program models of their mathematical and scientific knowledge and then share, discuss and modify the models through a Web-based system, WebReports. We try to ensure that the potential of collaboration is exploited in all its forms, by including asynchronous discussion and exchange around WebReports as part of any activity sequence, alongside synchronous interchange, both face-to-face and at a distance. We also aim for a more explicit promotion of learning mathematics through the processes of modeling and sharing, collective reflection and participation in a joint enterprise. (Hoyles 2008, p. 258)

This passage indicates a change from the type of research reported at ICMI events 20 years earlier, as there is a clear emphasis on the collaborative interaction, collective argumentation and cooperative meaning making that is afforded by the synchronous and asynchronous discussion made possible by Web-based networks. Hoyles goes on to acknowledge:

> With the benefit of hindsight, I now recognize that in my early Logo research, I may not have given sufficient attention to the complexities underlying the introduction of microworlds into institutionalized mathematics teaching, even microworlds that had been carefully designed in terms of computer tools, sequenced activities and the teacher's role. These complexities include a recognition of the ways the 'computer' shapes mathematical knowledge and the interactions between learners and between learners and teachers, and crucially, how computers frame the language in which mathematics is expressed and the meanings of 'doing math' and communicating mathematically. (Hoyles 2008, p. 259)

At the same congress, the report of Topic Study Group 15: *The Role and the Use of Technology in the Teaching and Learning of Mathematics* included a commentary by Healy and Kaput (2008) on a plenary lecture by Jim Kaput, which noted a similar recognition of the role of interaction in learning mathematics, and which indicated that this has become increasingly possible because of the affordances of Web-based technologies.

> Alongside the aspects of technology linked to its representational infrastructures, Jim Kaput, USA, brought into focus the communicational affordances of digital technologies. With advances in connectivity, he described how it is becoming possible for learners to interact alongside computational agents as well as other learners in mathematical explorations, bringing a new layer to what we understand by an experiential approach to learning mathematics—and another possibility with both epistemological and cognitive repercussions. (Healy and Kaput 2008, p. 357)

This new acknowledgement is even more evident in the study volume of the 17th ICMI study, *Digital Technologies and Mathematics Teaching and Learning: Rethinking the Terrain* (Hoyles and Lagrange 2009). Strong evidence for Lerman's "turn to the social" is found in the inclusion of an entire chapter in this study volume on social practices in learning mathematics through technology entitled *Technology, Communication, and Collaboration: Re-thinking Communities of Inquiry, Learning and Practice* (Beatty and Geiger 2009). The chapter traces the emergence of technology-mediated collaborative practices in mathematics classrooms, in both proximate and remote contexts, from the late 1980s until the time of the 17th ICMI symposium in 2006 and argues that this development paralleled the changing influence of different theories of learning over time, in particular, the theories generally grouped under the terms behaviorism, constructivism and socio-culturalism. These theories of learning, at different times, have influenced curriculum development and pedagogical approaches in school classrooms which have moved toward a greater recognition of the role of social interaction in learning and teaching. Beatty and Geiger (2009) see this influence reflected in mathematics education research and report that up to 31% of papers presented at the ICMI 17 symposium focused on the role of social interaction directly or included a theoretical perspective that could be considered social in origin, indicating a noteworthy level of interest in this perspective on mathematics learning.

Beatty and Geiger's (2009) chapter also detailed a topology for how different types of technology are designed or utilized to support collaborative learning communities. They argue that collaborative practices can be promoted through technologies designed for:

1. both learning mathematics and collaboration;
2. learning mathematics, but not specifically for collaboration;
3. collaboration, but not necessarily learning mathematics;
4. neither learning mathematics nor collaboration.

These authors point out that although the studies reviewed to develop the typography documenting the use of technology to facilitate collaborative practice in very different ways, there were characteristics common to each study.

> Some look at the discourse between two or three learners in front of a desktop computer, others the interaction of many users contributing to an online database. In all studies, the underlying theoretical frameworks emphasized the importance of discourse and collaboration as essential to the process of learning mathematics. All used rich open-ended tasks, and all specified the affordances of the particular kind of technology used for engendering collaborative communities of practice—whether based around aggregated dynamic representations, or archiving threads of discussion in student-managed discussion platforms. And in all studies, technology was viewed as a means of mediating social interaction. (Beatty and Geiger 2009)

Although Beatty and Geiger (2009) document a substantial corpus of literature in the area, they conclude that social theories of learning, such as socio-culturalism and social constructivism, were conceived before technology had any great influence on mainstream mathematics classrooms. While there is now increasing interest in technology-influenced collaborative practice, much research still remains to be undertaken before the role of technology in mediating collaboration can be genuinely integrated into social theories of learning.

### 3.4 The growth of interest in the social perspective on teaching and learning mathematics with technology

The discussion presented above documents a shift in interest among researchers toward theoretical perspectives, where social interaction is acknowledged as a key influence on students' intellectual development and technology is viewed as a powerful agent in facilitating collaborative learning practices. This interest has changed from the late 1980s where the role of social interaction when learning mathematics with the assistance of technology was only beginning to be considered. Twenty years later, however, the influence of technology-enhanced collaborative practices in teaching and learning is now a major area of activity within the field of mathematics education, although further research is needed into the potential offered by technology to mediate productive collaborative learning practices. Further, although there is now a considerable body of research literature in this field, there is still much work to be done in relation to the advantages of the affordances offered by ICTs to collaborative teaching and learning practices.

Has an understanding of the role of social interaction in learning influenced the development of technologies to support collaborative approaches to learning, or has the development of multi-user technologies that allow for synchronous and asynchronous interaction promoted more collaborative approaches to learning? Despite the developing literature base in both the roles of social interaction and of technology in promoting learning in mathematics, it is unclear which of these influences has led to changes in the other.

While there is evidence of a developing corpus of research literature in the field of technology-enhanced mathematics learning and teaching (Hoyles & Noss, 2003), it is less apparent that this research has influenced school classrooms. Despite long-term attention, the use of computers and other technologies in school mathematics classrooms has been restricted to date by economic, social and practical constraints (Guin & Trouche, 1999; Kaput, 1992; Kemp, Kissane & Bradley, 1996; Mariotti, 2002). As Mariotti observes, "the entry of computers into schools has been slow, and their integration in school practice even slower" (Mariotti, 2002, p. 720). None-the-less, given the growing trend within schools to connect their communities through technology, it is vital that the role digital technologies can play in promoting collaboration between learners and teachers be better understood and more completely theorized. This may, in turn, lead to a reconceptualization of teaching and learning in technology-rich contexts.

## 4 Learning mathematics in a Web-based social environment: mathematics as performance?

Given the growing interest in social interactions as a part of learning with technology, it is important to consider how we might reconceptualize the way we teach and learn with technology. Given the social and multimodal affordances of new media, which introduce 'audience' and 'multimodal communication', we suggest that one way to reconceptualize social interactions in learning with technology would be to view them through a performative lens.

What does "performance" have to do with the role of technology in mathematics education? Hughes (2008) notes that the Web is fast becoming a "performative medium". This is evident in the multimedia authoring tools used to create online content, such as Flash, which often use performance metaphors in their programming environment. For example, you program on what is referred to as the "stage", you use "scenes" to organize "actors" or "objects" and their relationships, and you control the performance using "scripts". Hughes (2008) suggests that the new media that is infusing the Web draws us into performative relationships with and representations of our "content" and in our relationships with others. To use new media is to in part adopt a performative paradigm.

The Web as a performative medium is evident in the success of portals such as YouTube. In fact, students, teachers and others have used YouTube to share mathematical performances. Along similar lines, we have seen the emergence of a Math Performance Festival [http://www.mathfest.ca] in Canada that was supported by the Fields Institute and the Canadian Mathematical Society. An example of a performance from the Festival is *Now I'm a Trapezoid* [available at http://www.edu.uwo.ca/mathscene/geometry/geo1.html], which is a song by a fifth grade student about a triangle that has lost its head. Saddened by this loss, the triangle laments that it is now a trapezoid (see Fig. 3). Does this view of shapes and their relationships make a difference: for students, for teachers, for you? How might the student, who sings the song, see triangles and trapezoids differently? How might he or she *feel* differently about these shapes and about mathematics in general? A second example from the Festival is *Measuring the Millimeters to You* [available at http://www.edu.uwo.ca/mathscene/pst/pst5.html], a song written and performed by elementary preservice teachers. In this romantic ballad, two friends are saddened because of the great distance (100,000 mm) that separates them (see Fig. 4).


**Fig. 3** A performance from the Math Performance Festival

**Fig. 4** A second performance from the Math Performance Festival

Then, they realize that 100,000 mm is the same as 10,000 cm. Also, when they divide by 10 and then by 10 again, they see that they are really not that far apart, only 100 m. This song provides insight into the metric system in ways that are humorous, emotional and difficult to forget. Do these songs offer opportunities to experience the new and the wonderful in mathematics? Do they offer mathematical surprise, emotional mathematical moments, or a sense of mathematical beauty? Such performances, where students are seen on video *celebrating* mathematical ideas, might perhaps help improve the public image of mathematics by helping us better appreciate mathematics as a creative, human endeavor.

It has been suggested that students might be viewed as "performance mathematicians" and that a performance (as in the Arts) lens might be useful in framing the teaching, learning and doing of mathematics (Gadanidis and Borba, 2008; Gadanidis et al. 2008), especially in a technology-rich setting. Such a lens helps us see and judge mathematics activity as we would see and judge a film. For example, if a mathematics activity was to be judged as we might judge a film, then Gadanidis and Borba (2008) (using the work of Boorstin 1990) suggest that it would 'work' if it offered us opportunities to experience the following pleasures: seeing the new and the wonderful in mathematics; being surprised mathematically; feeling emotional moments in doing and learning mathematics; sensing mathematical beauty. Our mathematics education culture does to some degree promote helping students experience the 'new and the wonderful' to the extent that our curriculum documents promote the making of connections and exploring relationships and getting a sense of the big ideas of mathematics. However, an emphasis on surprise, on emotional moments or on beauty would be a rare find in a curriculum document. How might these attributes expand our view of social interactions in mathematics learning with technology?

The case of the Math Performance Festival offers a glimpse into how collaboration in mathematics learning might be extended to include math performance, or perhaps how collaboration in a media-rich digital environment might be reconceptualized as collaborative performance. Wikis, which were originally text based, now have multimodal affordances and students and teachers can create videos using a video camera or a webcam and embed the videos in the wiki discussion (as discussed earlier in the case of the ICZ platform). Also, they might embed an existing video into the wiki, perhaps from Youtube or other video sharing platforms. Alternatively, they might use Comic Life (to give an example of one multimedia software package licensed by some school districts and ministries of education for use by schools) to create and share a math comic strip as an image or as a movie. Also, the collaborative editing of text that is available in a wiki is also becoming possible for multimedia content. For example, students and teachers have access to a variety of (freely available) screen capture and video editing software (like *Jing* and *Movie Maker*), which allows them to create new performances that include both new and recycled multimedia content. [For a sample of this type of remixing to create new content, see the "Where I'm from" video performance available at http://faculty.uoit.ca/hughes/clip1.html, which was created by a seventh grade student.]

## 5 A look to the future

The last two decades have seen a tremendous change in the technologies available in schools and in society in general. In mathematics education, we have seen two parallel shifts: (1) from a focus on computer-based learning to Web-based learning, and (2) from a focus on the individual learner to sociocultural aspects of learning. By tracing the emergence of both of these shifts, from the early 1980s to the present, through reference to the activities of ICMI, it seems that the emergence of Web 2.0, with its social and collaborative affordances, provided the opportunity for these two shifts to come together in a potentially complementary and powerful way. At the same time, we wonder what paradigm shift may be necessary for this potential to manifest itself. This shift will mean a change in the way we think about learning and teaching, how we judge achievement in mathematics and how we view mathematics itself.

Lankshear and Knobel (2006) suggest that new technologies bring with them new mindsets. The old mindset includes these characteristics:

- focus on individual intelligence;
- expertise and authority 'located' in individuals and institutions;
- space as enclosed and purpose-specific;
- social relations of 'bookspace'; a stable 'textual order (p. 38).

While the new mindset includes different characteristics:

- focus on collective intelligence;
- expertise and authority are distributed and collective: hybrid experts;
- space as open, continuous and fluid;
- social relations of emerging 'digital media space'; texts in change (p. 38).

Does the mindset associated with new Web 2.0 technologies spill over into education? We have provided evidence that there is now an established body of research within mathematics education, often drawn from authentic classrooms, which is consistent with this view. But, is there a similar change taking place in mainstream mathematics classroom? Lankshear and Knobel note that the day-to-day business of school is still dominated by conventional paradigms, with schools "reproducing familiar conventional literacies through the uses of new technologies." (p. 30). In addition, Sprague et al. (2007) point out "that so-called *early-adopters* of technology may have made up the majority of faculty and students who have so far been involved in the online education phenomenon" (p. 158).

Adopting the "new" mindset described by Lankshear and Knobel in mathematics education requires a complementary "new" mindset about mathematics itself and what it means to learn and know mathematics. Currently, learning mathematics, in many classrooms, is viewed as the private appropriation of personal knowledge. The argument we have presented above suggests that this need not be the case and that becoming mathematically "wise" can be a process that involves highly collaborative and public practices. Mathematical knowing, in this sense, is related to the co-construction of knowledge and a celebration of this sharing via participation in the "performance" of mathematics, not just its personal ownership.

For the most part, mathematics is today a classroom-based experience. What students do in mathematics classrooms typically does not spill out to their world around them. Students do not typically go home and say "let me show you what we did in math today, it is so exciting…". They do not share their math experiences and ideas the way they might with a good book or a favorite movie, for example. Parents, and other adults, do not typically hesitate to express negative views of mathematics. How do we imagine a subject as ostracized, authoritative and feared or disliked as mathematics becoming a space for open, fluid collaboration?

New technologies offer opportunities for bringing mathematics into public light, engaging the complex, interesting and human side of mathematics, and uniting people to experience mathematics as socially constructed. The Math Performance Festival described above is one example that helps bring the mathematical ideas of students into public forums where is can be shared and critiqued and which then provides opportunity for the continued development of knowledge and understanding within a supportive community of learners.

The notion of working within a supportive learning community is exemplified by mathematician, Megumi Harada (2008), in an interview as part of the *Windows into Elementary Mathematics Project* conducted by the Fields Institute:

> I love mathematicians … When I was a university student, I was in love with a lot of things … I studied literature … anthropology … linguistics … philosophy … East Asian Studies … I knew a whole lot of people as a young student and I can say, without any doubt, the math students were the most fun to be around … I think it's because, as a group, mathematicians love what they do more than many, many groups of people I know … I find it very difficult to imagine for myself doing math all the time just by myself … I just can't imagine not being in a group of people.

Harada expresses a very social view of what it is to do mathematics and it appears (for her) that working and sharing with others *is* doing mathematics. This is not a view that is held in practice in many school classrooms, even those who have access to technologies, which would make wider collaboration possible. New technologies provide the opportunity to connect learners in ways we have only begun to explore, but it is not the technology itself that ensures the connection of learners, but how the technology is used. As argued by Beatty and Geiger (2009), a wide variety of technologies can mediate interaction, but it is how these technologies are used to support collaborative practices that makes the difference. The use of technologies to support collaborative learning practices requires new ways of learning and teaching and a new mindset about what it is to do mathematics.

For the "new" mindset described by Lankshear and Knobel to manifest itself in mathematics education, it requires not only new technologies, but also a new view of learning and doing mathematics, or rather a new view of *performing* mathematics.

## References


Afanasiev, A., & Crowe, D. (2008). Current problems and challenges in distance teaching and learning. In M. Niss (Ed.), *ICME-10 Proceedings* (pp. 534–537). Denmark: Roskilde University.

Beatty, R., & Geiger, V. (2009). Technology, communication and collaboration: Re-thinking communities of inquiry, learning and practice. In C. Hoyles, & J.-B. Lagrange (Eds.), *Digital technologies and mathematics teaching and learning: Rethinking the terrain*. New York: Springer.

Beatty, R., & Moss, J. (2006). Connecting grade 4 students from diverse urban classrooms: virtual collaboration to solve generalizing problems. In C. Hoyles, J.-B. Lagrange, L. Hung Son, & N. Sinclair (Eds.), *Technology revisited: proceedings of the seventeenth study conference of the International Commission on Mathematical Instruction* (pp. 27–36). Hanoi: Hanoi Institute of Technology.

Bold, M. (2006). Use of wikis is graduate course work. *Journal of Interactive Learning Research, 17*(1), 5–14.

Boorstin, J. (1990). *The hollywood eye: what makes movies work*. New York: Harper Collins Publishers.

Borba, M. C. (2009). Potential scenarios for Internet use in the mathematics classroom. *ZDM, 41*(4), 453–465.

Borba, M., & Gadanidis, G. (2008). Virtual communities and networks of practising mathematics teachers: The role of technology in collaboration. In T. Wood (Series Editor), & K. Krainer (Volume Editor), *International handbook of mathematics teacher education, Vol. 3: Participants in mathematics teacher education: individuals, teams, communities, and networks* (pp 181–209). Rotterdam: Sense Publishers.

Borba, M. C., & Villarreal, M. E. (2005). *Humans-with-media and the reorganization of mathematical thinking*. New York: Springer.

Carss, M. (1986). *Prodeedings of the 5th international conference on mathematics education*. Boston: Birkhauser.

Churchhouse, R. F. (1986). *The influence of computers and informatics on mathematics and its teaching*. London: Cambridge University Press.

Cobb, P., & Bauersfeld, H. (1995). *The emergence of mathematical meaning: Interaction in classroom cultures*. Hillsdale, NJ: L. Erlbaum Associates.

Drijvers, P., & Stacey, K. (2008). Technology in mathematics education. In M. Niss (Ed.), *ICME-10 Proceedings* (pp. 281–286). Denmark: ICMI.

Fey, J. T. (1993). Technology and mathematics education at ICME-7. In J. A. Dossey (Ed.), *American perspectives on the seventh international congress on mathematical education* (pp. 6–11). Reston, VA: National Council of Teachers of Mathematics.

Fraser, R., Meissner, H., Ralston, T., Roseveare, D., & Mohyla, J. (1986). In M. Carss (Ed.), *Proceedings of the 5th international conference on mathematics education* (pp. 159–176). Boston: Birkhauser.

Gadanidis, G., & Borba, M. (2008). Our lives as performance mathematicians. *For the Learning of Mathematics, 28*(1), 44–51.

Gadanidis, G., Hughes, J., & Borba, M. (2008). Students as performance mathematicians. *Mathematics Teaching in the Middle School, 14*(3), 168–175.

Gadanidis, G., Jardine, R., & Sedig, C. (2007). Designing digital windows into mathematics. In C. Montgomerie & J. Seale (Eds.), *Proceedings of ED-Media 2007* (pp. 3532–3537). Cheseapeake, VA: Association for the advancement of computing in education.

Gaulin, C., Hodgson, B. R., Wheeler, D. H., & Egsgard, J. C. (Eds.). (1994). *Proceedings of the 7th international congress on mathematical education*. Ainte-Foy, Quebec: Presses de l'Universite Laval.

Geiger, V. (2005). Master, servant, partner and extension of self: A finer grained view of this taxonomy. In P. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonough, R. Pierce, & A. Roche (Eds.), *Building connections: Theory, research and practice, proceedings of the 28th annual conference of the Mathematics Education Research Group of Australia* (pp. 369–376). Sydney: MERGA.

Geiger, V. (2006). More than tools: mathematically enabled technologies as partner and collaborator. In C. Hoyles, J.-B. Lagrange, Le. Hung Son, & N. Sinclair (Eds.), *Technology revisited: proceedings of the seventeenth study conference of the international commission on mathematical instruction* (pp. 182–189). Hanoi: Hanoi Institute of Technology.

Goos, M., Galbraith, P., & Renshaw, P. (1999). Establishing a community of practice in a secondary mathematics classroom. In L. Burton (Ed.), *Learning mathematics: From hierarchies to networks* (pp. 36–61). London: Falmer Press.

Goos, M., Galbraith, P., Renshaw, P., & Geiger, V. (2000). Reshaping teacher and student roles in technology-enriched classrooms. *Mathematics Education Research Journal, 12*(3), 303–320.

Goos, M., Galbraith, P., Renshaw, P., & Geiger, V. (2003). Perspectives on technology mediated learning in secondary school mathematics classrooms. *Journal of Mathematical Behavior, 22*(1), 73–89.

Grant, L. (2006). Using wikis in school: A case study. *Futurelab*. http://www.futurelab.org.uk/resources/publications_reports_articles/discussion_papers/Discussion_Paper258/. Accessed 12 June 2007.

Guin, D., & Trouche, L. (1999). The complex process of converting tools into mathematical instruments: The case of calculators. *International Journal of Computers for Mathematical Learning, 3*(3), 195–227.

Harada, M. (2008). Spherical geometry: Do parallel lines meet? *Fields Institute*. http://www.fields.utoronto.ca/mathwindows/sphere/. Accessed 1 October 2008.

Healy, L., & Kaput, J. (2008). The role and use of technology in the teaching and learning of mathematics. In M. Niss (Ed.), *Proceedings of the 10th international congress on mathematics education* (pp. 355–358). Denmark: Roskilde University.

Hirst, A., & Hirst, K. (Eds.). (1988). *Proceedings of the sixth international congress on mathematical education*. Budapest: Janos Bolyai Mathematical Society.

Hoyles, C. (2008). Reflections and transformations: A mathematical autobiography. In M. Niss (Ed.), *Proceedings of the 10th international congress on mathematics education* (pp. 255–265). Denmark: Roskilde University.

Hoyles, C., & Lagrange, J.-B. (2009). *Digital technologies and mathematics teaching and learning: Rethinking the terrain*. New York: Springer.

Hoyles, C., Lagrange, J.-B., Le Hung, Son., & Sinclair, N. (Eds.). (2006). *Technology revisited: proceedings of the seventeenth study conference of the international commission on mathematical instruction*. Hanoi: Hanoi Institute of Technology.

Hoyles, C., & Noss, R. (2003). What can digital technologies take from and bring to research in mathematics education? In A. J. Bishop (Ed.), *Second international handbook of mathematics education* (pp. 323–349). Dordrecht: Kluwer.

Hughes, J. (2008). The performative pull of research with new media. *International Journal of Qualitative Methods, 7*(2), 16–34.
Jarvis, D. H. (2006). On-line professional development for mathematics educators: Overcoming significant barriers to the modelling of reform-oriented pedagogy. In C. Hoyles, J.-B. Lagrange, Le. Hung Son, & N. Sinclair (Eds.), *Technology revisited: proceedings of the seventeenth study conference of the international commission on mathematical instruction* (pp. 246–252). Hanoi: Hanoi Institute of Technology.
Kaput, J. (1992). Technology in mathematics education. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 515–556). New York: Macmillan.
Kemp, M., Kissane, B., & Bradley, J. (1996). Graphics calculator use in examinations: Accident or design? *Australian Senior Mathematics Journal, 10*(1), 36–50.
Knight, G. (1994). Mathematics in distance learning. In C. Gaulin, B. R. Hodgson, D. H. Wheeler, & J. C. Egsgard (Eds.), *Proceedings of the 7th international congress on mathematics education* (pp. 211–225). Ainte-Foy, Quebec: Presses de l'Universite Laval.
Landow, G. P. (2006). *Hypertext 3.0. Critical theory and new media in a era of globalization*. Baltimore, MD: The John Hopkins University Press.
Lankshear, C., & Knobel, M. (2006). *New literacies: Everyday practices and classroom learning* (2nd ed.). London: Open University Press.
Lankshear, C., & Knobel, M. (2007). Sampling "the new" in new literacies. In M. Knobel & C. Lankshear (Eds.), *A new literacies sampler* (pp. 1–24). New York: Peter Lang.
Lerman, S. (2000). The socio-cultural turn in studying the teaching and learning of mathematics. In H. Fujita, Y. Hashimoto, B. Hodgson, P. Y. Lee, S. Lerman, & T. Sawada (Eds.), *Proceedings of the ninth international congress on mathematical education* (pp. 157–158). Tokyo: Kluwer.
Levy, P. (1997). *Collective Intelligence*. New York: Plenum.
Li, J., & Wisenbaker, J. M. (2008). Research and development in the teaching and learning of probability and statistics. In M. Niss (Ed.), *Proceedings of the 10th international congress on mathematics education* (pp. 337–340). Denmark: Roskilde University.
Lichtenberg, B. K. (1993). The impact of the calculator on the elementary school curriculum: Reflections on ICME-7. In J. A. Dossey (Ed.), *American perspectives on the seventh international congress on mathematical education* (pp. 19–20). Reston, VA: National Council of Teachers of Mathematics.
Mandelbrot, B. B. (1994). Fractals, the computer, and mathematics education. In C. Gaulin, B. R. Hodgson, D. H. Wheeler, & J. C. Egsgard (Eds.), *Proceedings of the 7th international congress on mathematics education* (pp. 77–98). Ainte-Foy, Quebec: Presses de l'Universite Laval.
Mariotti, M. (2002). The influences of technological advances on students' mathematics learning. In L. D. English (Ed.), *Handbook of international research in mathematics education* (pp. 695–723). Mahwah, NJ: Lawrence Erlbaum.
Mayo, J. (1993). Just one more computer, please. In J. A. Dossey (Ed.), *American perspectives on the seventh international congress on mathematical education* (p. 18). Reston, VA: National Council of Teachers of Mathematics.
Niss, M. (Ed.). (2008). *Proceedings of the 10th international congress on mathematics education*. Denmark: Roskilde University.
Noddings, N. (1990). Constructivism in mathematics education. In R. Davis, C. Maher, & N. Noddings (Eds.), *Constructivist views on the teaching and of learning of mathematics* (pp. 7–18). Reston, VA: National Council of Teachers of Mathematics.
Pea, R. (1985). Beyond amplification: Using the computer to reorganize mental functioning. *Educational Psychologist, 20*(4), 167–182.
Pea, R. (1993a). Learning scientific concepts through material and social activities: Conversational analysis meets conceptual change. *Educational Psychologist, 28*(3), 265–277.
Pea, R. (1993b). Practices of distributed intelligence and designs for education. In G. Salomon (Ed.), *Distributed cognitions: Psychological and educational considerations* (pp. 47–87). Cambridge: Cambridge University Press.
Richardson, W. (2006). *Blogs, wikis, Podcasts and other powerful Web tools for classrooms*. Thousand Oaks, California: Corwin Press.
Schrage, M. (2001). The relationship revolution. http://web.archive.org/web/20030602025739/http://www.ml.com/woml/forum/relation.htm. Accessed 12 July 2008.
Shumway, R. (1989). Technology, mathematics and the international congress. In T. Cooney (Ed.), *American perspectives on the sixth international congress on mathematical education* (pp. 15–20). Reston, VA: National Council of Teachers of Mathematics.
Sloan, M., & Olive, J. (2006). Distance learning: mathematical learning opportunities for rural schools in the United States. In L. Son, N. Sinclair, J.-B. Lagrange, & C. Hoyles (Eds.), *Technology revisited: proceedings of the seventeenth ICMI study conference* (pp. 518–523). Hanoi: Hanoi Institute of Technology.
Sprague, D., Maddux, C., Ferdig, R., & Albion, P. (2007). Online education: Issues and research questions. *Journal of Technology and Teacher Education, 15*(2), 157–166.